Emergent Behaviors of Protector, Refugee, and Aggressor Swarms

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***Abstract*—Simple rules, when executed by individual agents in a large group, or swarm, can lead to complex behaviors that are often difficult or impossible to predict knowing only the rules. However, aggregate behavior is not always unpredictable—even for swarm models said to be beyond analysis. For the class of swarming algorithms examined herein, we analytically identify several possible emergent behaviors and their underlying causes: clustering, drifting, and explosion. They also analyze the likelihood of these behaviors emerging from randomly selected swarm configurations and present a few examples. The analytic results are illustrated via simulation.**

***Index Terms*—Dynamic systems, emergent behavior, swarm intelligence.**

I. Introduction

Often, determination of emergent behavior from simple rules of interaction in swarm intelligence escapes both analytic and intuitive inspection. Swarm intelligence, with important applications in telecommunications [9], [15], business [4], robotics [3], [13], and optimization [10], [11], makes use of a plurality of highly disjoint agents interacting using simple rules. Simple swarm algorithms have been employed to assist with load balancing of peer-to-peer networks [18], routing within mobile *ad hoc* radio networks [12], and self-organizing construction and assembly [17].

In all but the simplest cases, however, it remains extremely difficult to predict with certainty the emergent behavior of the swarm as a whole. Bonabeau and Meyer [4] initially posed a set of problems to illustrate how simple rules, when applied by many individual agents simultaneously, can result in a complex and sometimes unpredictable emergent behavior. Designing specific collective behaviors by specifying the appropriate individual behaviors is a nascent research subject [5], [6], [8], and in these works researchers sometimes turn to the "canonical" problem discussed here, or a close variant.

Certain properties of swarms, such as clustering and subgrouping, have historically been illustrated by simulation; however, such properties have previously not been predictable or readily explained. In this correspondence, we analytically describe certain swarm behaviors by assigning a particular dynamical interpretation to the swarm rules and examining the properties of the corresponding dynamics. Our case study derives from a swarming model designed for Icosystem, Inc. [14], that implements the initial suggestions of Bonabeau and Meyer. Regarding their model, they state, "Although predicting the group's collective behavior is a task beyond human grasp, it can often be done using simulation modeling." Later, using mainly geometric arguments, certain macroscopic behaviors were in fact deduced for the Icosystem model [1]. However, without reference to the dynamics that drive these behaviors, little was known about when and why they occur. Here, we show that Icosystem-type swarms can cluster, drift, and explode, and we give conditions under which these behaviors may or may not occur. The following analysis represents a significant step beyond computer simulation and reveals previously unidentified causes for certain observed behaviors. As importantly, the dynamical and graphical techniques employed here may be extensible to other types of swarming systems.

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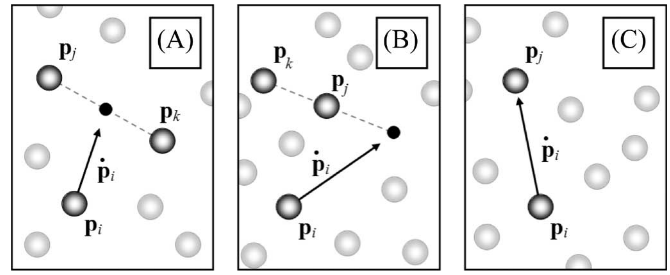

Fig. 1. Two principle swaming modes, (A) and (B), illustrating "protector" and "refugee" behavior, and a third special case (C) illustrating "aggressor" behavior.

To begin, we first note the simple rules that each swarm agent will enact, as per the Icosystem model.

A. *Protector:* Every agent in the swarm picks two others and tries to position itself between them.
B. *Refugee:* Every agent in the swarm picks two others and tries to position itself so that the first is directly between itself and the second.

These are illustrated in Fig. 1. To be meaningful, we must first translate these rules into some kind of dynamical equations. One simple choice appears to be a system in which every agent is attracted to its desired location with an attraction that is negatively proportional to the distance from that location. For instance, in scenario (A), if a protector i wishes to place itself between an aggressor j and a refugee k, then it might employ the formula

$$\dot{\mathbf{p}}_i = -\gamma\left(\mathbf{p}_i - \frac{\mathbf{p}_k + \mathbf{p}_j}{2}\right) \tag{1}$$

where γ is an arbitrary positive constant and $\mathbf{p}_i(t) = [x_i^{(1)}(t), x_i^{(2)}(t), \ldots, x_i^{(n)}(t)]^{\mathrm{T}}$ is an n-dimensional Cartesian position. To avoid confusion between vector and scalar quantities, vectors are written in bold. Matrices are capitalized. Time derivatives are indicated by superscript dot. Similarly, in (B), if refugee i wishes to place protector j between itself and aggressor k, the formula would be

$$\dot{\mathbf{p}}_i = -\gamma\left(\mathbf{p}_i - (\mathbf{p}_k - 2\mathbf{p}_j)\right). \tag{2}$$

In fact, both rules can be viewed as special cases of the general formula

$$\begin{aligned}\dot{\mathbf{p}}_i &= -\gamma\left(\mathbf{p}_i - (\alpha\mathbf{p}_j + \beta\mathbf{p}_k)\right)\\ &= \gamma\left(-\mathbf{p}_i + \alpha\mathbf{p}_j + \beta\mathbf{p}_k\right) \qquad \text{with } \alpha + \beta = 1.\end{aligned} \tag{3}$$

In other words, agent i's velocity is aimed at a point along the line connecting agents j and k. The parameter γ will affect the speed at which individual agents move but not the overall behavior of the swarm; without loss of generality we may set $\gamma = 1$. Thus, "protector" swarms are implemented if $0 < \alpha < 1$, and "refugee" swarms are implemented if either $\alpha > 1$ or $\beta > 1$. Furthermore, we also see the emergence of another rule, corresponding to $\alpha = 1$ or $\beta = 1$, which states

C. *Aggressor:* Every agent in the swarm picks another and chases it.

This is also illustrated in Fig. 1. All three rules attempt to position agent i somewhere on the line defined by the positions of j and k. Looking at the system as a whole, all of the agents' $x^{(1)}$ coordinates now evolve as

$$\dot{\mathbf{x}} = A\mathbf{x} \quad \mathbf{x} \in \mathbb{R}^n \quad A \in \mathbb{R}^{n\times n} \tag{4}$$

and similarly for their $x^{(2)}$ coordinates, etc., where $\mathbf{x} = [x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}]$. The state matrix A might look, for example, like

$$A = \begin{bmatrix} -1 & \beta & \alpha & & & \\ & -1 & \alpha & & \beta & \\ \alpha & \beta & -1 & & & \\ & & & \ddots & & \\ & \beta & & & \alpha & -1 \end{bmatrix} \tag{5}$$

and will always have diagonal entries equal to -1.

We note that (3) is actually a kinematic equation, making no reference to particle accelerations or forces. In the aggregate, however, (4) is commonly termed a linear "dynamic" equation. Thus, we will adopt the "dynamic" terminology henceforth.

II. Swarm Behaviors

There are several observations we can make at this point. First, A always has a nonzero nullspace, spanning at least one dimension given by the null vector

$$\mathbf{v}_1 = [1, 1, 1, \ldots, 1]^{\mathrm{T}}. \tag{6}$$

In other words, each row of A must add to zero. Second, by Gerschgorin's theorem[1] we may restrict the possible locations of A's eigenvalues to a circle in the complex plane centered at -1 with radius

$$r = |\alpha| + |\beta|. \tag{7}$$

Note that at least one eigenvalue will reside at the origin (corresponding with the eigenvector $\mathbf{v}_1$). Third, there must be at least one zero eigenvalue associated with the eigenvector $\mathbf{v}_1$. Even if $\mathbf{v}_1$ is the only null vector, more zero eigenvalues are possible if they are simply degenerate [7]. Eigenvalues and eigenvectors are of fundamental importance in what follows; eigenvalues with negative real parts suggest stable behavior in some sense (opposite for positive real parts), but zero eigenvalues require closer examination. The eigenvector $\mathbf{v}_1$ can be viewed simply as the point of dynamic equilibrium: if the system dynamic tends toward, or reaches $\mathbf{v}_1$, then all movement will cease.

Perhaps one of the most relevant questions about swarm behavior in this context is the question of the cohesion of the swarm. Some Icosystem swarms have been observed to cluster, while others have been observed to disperse. Mathematically, these are very similar to questions of stability and convergence, and with the system model and observations given above we may begin to gain insight into the question of swarm cohesion. If we assume that $\mathbf{v}_1$ is the only null vector, then the solution transition matrix may be written in terms of its Jordan normal form as

$$\Phi(t) = e^{At} = V e^{Jt} V^{-1} \quad J = \begin{bmatrix} [J_1] & 0 \\ 0 & [J_2] \end{bmatrix}$$

$$J_1 = \begin{bmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & \ddots & 1 \\ & & & 0_m \end{bmatrix} \quad J_2 = \begin{bmatrix} \lambda_{m+1} & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}. \tag{8}$$

Note the possibility in J_1 of multiple degenerate zero eigenvalues. (While some eigenvalues in J_2 may be repeated, we neglect this possibility because the sign of their real parts determines whether they dominate the dynamics, not their algebraic multiplicity.) Exponentiating J_1 will yield an upper triangular transition matrix with entries of the form $(1/k!)t^k$ [7, pp. 297 and 298]. Given initial position $\mathbf{x}_0$, we define the product $V^{-1}\mathbf{x}_0 =: \mathbf{w}$ and note then that the system solution is given by

$$\begin{aligned} \mathbf{x}(t) &= \Phi(t)\mathbf{x}_0 \\ &= w_1\mathbf{v}_1 + w_2(\mathbf{v}_1 t + \mathbf{v}_2) + w_3\left(\mathbf{v}_1\frac{t^2}{2} + \mathbf{v}_2 t + \mathbf{v}_3\right) \\ &\quad + \cdots + w_m\left(\sum_{i=1}^{m} \mathbf{v}_i \frac{t^{m-i}}{(m-i)!}\right) + w_{m+1}\mathbf{v}_{m+1}e^{\lambda_{m+1}t} \\ &\quad + \cdots + w_n\mathbf{v}_n e^{\lambda_n t}. \end{aligned} \tag{9}$$

Several possible behaviors now become apparent. One possibility is that the nonzero eigenvalues may have positive real parts and the swarm will exponentially explode. Even if the real parts are negative, it is still possible to see a "polynomial explosion" from terms on line two of (9). However, this will only occur if there are at least three zero eigenvalues $m = 3$. If there is only one, then the swarm will actually "cluster" (converge) at the location w_1 because $\mathbf{v}_1 = [1, 1, 1, \ldots, 1]^{\mathrm{T}}$. This implies that all agents' coordinates are now identical. If there are two zero eigenvalues, the swarm may exhibit "drifting" behavior. In this case, after the exponential transients have died, the agents will be left moving along the trajectory $(w_2 t + w_1) + w_2\mathbf{v}_2$. They may not converge but they will not separate any further from each other, either; they are all moving at constant speed in the same direction.

Any of these behaviors is generally possible with swarms executing "refugee" rule (B), because eigenvalues may exist in the right-hand complex plane (the Gerschgorin radius is $1 < r \leq 3$). It seems, in fact, that random initial configurations of rule (B) are divergent more often than not, especially for large n, a fact evidenced by statistics presented in the next section.

However, for the "protector" and "aggressor" rules the Gerschgorin radius is $r = 1$ so exponential explosion is not possible. Thus, the swarm will cluster to a single point unless simply degenerate zero eigenvalues occur in the system matrix A or there are additional null vectors. The following arguments explore the eigenvalue/eigenvector problem for "protector" and "aggressor" swarms in more detail. To begin, we first define the notion of a connected swarm by observing that the swarm can be represented by a digraph. If agent i uses the positions of agents j and k to define its motion in (3), then edges emanate from node i to nodes j and k in the associated digraph; see Fig. 4, for example. Thus, we naturally define a swarm to be weakly connected, or simply connected, if its associated digraph is weakly connected (there is a path from every node to every other node, regardless of the edge directions). Similarly, a swarm is strongly connected if its digraph is strongly connected (there is a path from every node to every other node through the directional edges).

[1]Gerschgorin's theorem [19] states that each eigenvalue of a real $n \times n$ matrix A will fall within one of the n circles in the complex plane with center $c = A_{kk}$ and radius $r = \sum_{i=1, i\neq k}^{n} |A_{ki}|$.

For a summary of the behaviors exhibited by the three types of swarms, readers may skip to the first paragraph of the conclusions.

Theorem 1: If a swarm under rules (A) or (C) is strongly connected, then its agentswill converge to a single point.

Proof: Before continuing, we recall the definition of a row-stochastic Markov matrix (a matrix with nonnegative entries whose rows sum to 1) and a positive Markov matrix (a Markov matrix with strictly positive entries). Though Markov matrices are frequently associated with stochastic processes, we use them in another capacity here. Note that we refer interchangeably to row-stochastic Markov matrices as simply Markov matrices.

Claim 1: The matrix exponential e^A is a Markov matrix.

To see this, note that $\tilde{A} := A + I$ is a Markov matrix. Thus

$$e^A = e^{\tilde{A}-I} = \frac{e^{\tilde{A}}}{e} \frac{I + \tilde{A} + \frac{1}{2}\tilde{A}^2 + \frac{1}{6}\tilde{A}^3 \cdots}{e}. \tag{10}$$

Using the fact that, since $\tilde{A}$ is a Markov matrix, $\tilde{A}^k$ is as well, it follows that e^A has only nonnegative entries. Furthermore

$$\begin{aligned} e^A \mathbf{v}_1 &= \frac{I + \tilde{A} + \frac{1}{2}\tilde{A}^2 + \frac{1}{6}\tilde{A}^3 \cdots}{e} \mathbf{v}_1 \\ &= \frac{\mathbf{v}_1 + \mathbf{v}_1 + \frac{1}{2}\mathbf{v}_1 + \frac{1}{6}\mathbf{v}_1 + \cdots}{e} = \mathbf{v}_1 \end{aligned} \tag{11}$$

ensuring that all rows sum to 1. Note the claim also follows from the observation that A is a particular type of continuous-time Markov generator.

Claim 2: The matrix exponential e^A is positive.

Assume the entry $[e^A]_{i,j}$ is zero. It follows from the Taylor series above that $[\tilde{A}^k]_{i,j} = 0$ for all positive integers k. Thus, it is not possible to jump from state i to state j in the Markov chain and the associated state graph is not strongly connected. This contradicts the theorem's requirement, however, so it must be that $[e^A]_{i,j} > 0$.

Claim 3: The transition matrix has the property that $\Phi(t) \to M$ as $t \to \infty$, where each row of M is identical.

It is known that $e^{Ak} \to M$ as $k \to \infty$ for integers k [2], because e^A is a positive Markov matrix. Since the Gerschgorin circles of A admit complex system eigenvalues only with negative real parts, any oscillatory entries of e^{At} must die out. The claim then follows by examining the convergence of e^{At} at integer values of t.

The proof is completed by noting that the steady-state system solution is now

$$\Phi(t)\mathbf{x}_0 \to M\mathbf{x}_0 = w_1 \mathbf{v}_1 \tag{12}$$

where w_1 is a real constant defined in (9) An identical argument can be presented for all n dimensions in which the swarm operates, thus proving the theorem. ■

We note that Theorem 1 necessarily implies that A must have one unique eigenvalue at zero, and consequently one unique null vector. The conditions of Theorem 1 are actually somewhat restrictive; for example, for "aggressor" swarms, the only strongly connected digraph with n nodes and n edges is a ring. Fortunately, there is a stronger result for "aggressor" swarms.

Corollary 1: If a swarm under rule (C) is connected, it will converge to a single point.

Proof: The proof follows from the observation that a rule-(C) system matrix A has the form of the (transpose of) a digraph incidence matrix. It is known [20] that the incidence matrix of a connected graph has $\operatorname{rank}(A) = n - 1$. Thus, A has one nonzero equilibrium vector (any multiple of $\mathbf{v}_1$). Assume the A has at least two simply degenerate zero eigenvalues. Then, in steady state after any oscillatory motion has damped, it will either converge to the equilibrium coordinates $w_1\mathbf{v}_1$ or drift. If the agents are drifting, then at least one agent must be moving away from all others, which violates rule (C). Thus, the only steady-state solution is convergence to the equilibrium point. ■

We conclude this section with a few comments. One might ponder whether the lesser condition of weak connectedness can be extended to Theorem 1. Unfortunately, a counterexample proves this is not possible: it is straightforward to see that there is a two-dimensional null space for the following rule-(A) system matrix A:

$$A = \begin{bmatrix} -1 & 0.5 & 0.5 & & & & \\ 0.5 & -1 & 0.5 & & & & \\ 0.5 & 0.5 & -1 & & & & \\ & 0.5 & & -1 & & & 0.5 \\ & & & & -1 & 0.5 & 0.5 \\ & & & & 0.5 & -1 & 0.5 \\ & & & & 0.5 & 0.5 & -1 \end{bmatrix}. \tag{13}$$

Here, agent 4 is not strongly connected with the others. Furthermore, the converse of Theorem 1—that a weakly connected (A) or (C) swarms will not converge—is not true because the following example gives a rule-(A) swarm that converges without being strongly connected

$$A = \begin{bmatrix} -1 & & 0.5 & 0.5 \\ & -1 & 0.5 & 0.5 \\ 0.5 & & -1 & 0.5 \\ 0.5 & & 0.5 & -1 \end{bmatrix}. \tag{14}$$

Lastly, we note that the proof of Theorem 1 actually encompasses a broad class of Icosystem-type swarms that are more general than either rule (A) or (C). Specifically, without modification it proves the convergence of any strongly connected swarm in which a given agent propels itself toward an arbitrary convex combination of the other agents' positions. Thus

$$A = \begin{bmatrix} -1 & & & 1 \\ 0.2 & -1 & 0.8 & \\ 0.5 & & -1 & 0.5 \\ 0.1 & 0.8 & 0.1 & -1 \end{bmatrix} \tag{15}$$

represents a swarm that will cluster to a single point.

III. Computer Experiments

The analysis and discussion above has yielded insight into several possible swarm behaviors (clustering, drifting, and explosion), and limited the possible behavior of the three cases in question. Since only "refugee" swarms will exhibit nonclustering behavior, we thought it interesting to randomly generate many different A matrices and examine their properties to get a feel for how frequently the different behaviors might occur. For each case, a computer generated 10 000 system matrices ranging in size from $n = 4$ to $n = 20$. (For $n = 3$ exhaustive search was applied rather than random assignment.) The diagonals of each matrix were fixed at -1; then, by uniform random probability, the computer picked two nondiagonal locations on each row. In one location, the number α was inserted, in the second the number $1 - \alpha$. Ten thousand matrices were generated and analyzed for each of four α values: $\alpha \in \{1.50, 1.75, 2.00, 2.25\}$. These values were selected because a fundamental change in the behavior of the simple three-agent system occurs when $\alpha \geq 2$. A real eigenvalue was deemed to be nonzero if its magnitude was greater than 10^{-8} after normalizing the largest magnitude eigenvalue to unity. The same rule was applied to determining if the real part of a complex eigenvalue was zero or not. Matrices of nonstrongly connected swarms were discarded.

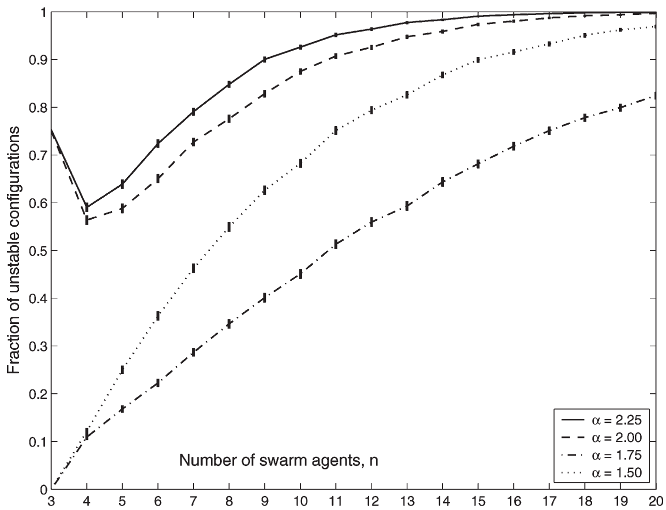

Fig. 2. Percentages of unstable swarm configurations for "refugee" swarms. The vertical bars on each plot are 95% confidence intervals [16].

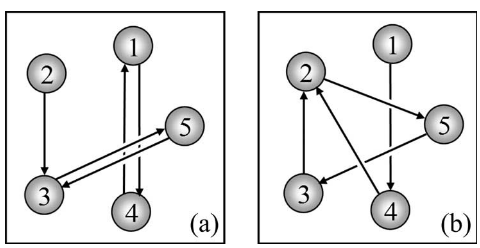

Fig. 3. (a) Nonconnected five-agent swarm and (b) a connected one. An arrow from j to k indicates that j is "chasing" k.

The computer looked for eigenvalues with positive real parts (indicating exponential explosion), or multiple zero eigenvalues (indicating possible drift or explosion). Contrary to the prevailing opinion about "refugee" swarms, Fig. 2 illustrates that stable-clustering behavior is possible, but increasingly unlikely as the number of swarm agents grows. At 20 agents ($\alpha = 2$), 9967 of 10 000 configurations were unstable (prone to drift or explode). It is also interesting to note that, as α increases (and $\beta = 1 - \alpha$ decreases) the likelihood of an unstable configuration increases as well. This is mathematically explained because, as the Gerschgorin radius $r = |\alpha| + |\beta|$ grows, a greater fraction of the Gerschgorin circle lies in the unstable right-hand plane.

IV. Examples

We now present a few example configurations and note their properties. For example, consider the following two system matrices for "aggressors:"

$$A_1 = \begin{bmatrix} -1 & & & 1 & \\ & -1 & 1 & & \\ & & -1 & & 1 \\ 1 & & & -1 & \\ & & 1 & & -1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & & & 1 & \\ & -1 & & & 1 \\ & 1 & -1 & & \\ & 1 & & -1 & \\ & & 1 & & -1 \end{bmatrix}. \quad (16)$$

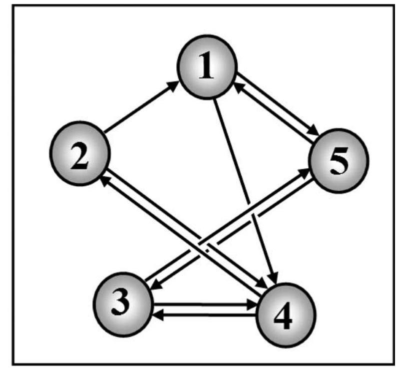

Fig. 4. Five-agent "refugee" swarm. Arrows indicate relationships, e.g., arrows from i to j and k indicate that i is trying to place j between itself and k (or vice versa).

Empty entries are zero. In what follows, we implement the swarm dynamics in two dimensions. As there is no dimensional cross coupling, the x and y dynamics are independent. The eigenvvalues of A_1 are $\{-1, 0, 0, -2, -2\}$, suggesting a drifting behavior. However, A_1 represents a nonconnected swarm, as easily shown in the Fig. 3(a). On the other hand, A_2 has eigenvalues $\{-1, -1, -1.5 \pm 0.866j, 0\}$, representing a swarm that will cluster, shown in Fig. 3(b).

The "refugee" case presents the most interesting behaviors. Consider the system represented by A_3

$$A_3 = \begin{bmatrix} -1 & & & 2 & -1 \\ -1 & -1 & & 2 & \\ & & -1 & 2 & -1 \\ & -1 & 2 & -1 & \\ -1 & & 2 & & -1 \end{bmatrix}. \quad (17)$$

The eigenvalues are $\{0, 0, -1, -2 \pm j\}$, with the zeros being simply degenerate. This swarm (Fig. 4) will generally drift after the initial transient movements die away. However, it is possible to arrange the swarm agents with initial positions that do not induce drifting, such as

$$\mathbf{x}_0 = \mathbf{y}_0 = [0.4173, 0.8166, 0.3561, -0.0970, -0.1509]^{\mathrm{T}}. \quad (18)$$

Looking back at (9), a Jordan decomposition gives $\mathbf{w} = V^{-1}\mathbf{x}_0 = [0, 0, 0.0153, 0.0307 \pm 0.4837j]$; therefore, the only terms remaining in (9) are the decaying exponentials and the swarm will cluster. Figs. 5–7 illustrate the phenomena of clustering and drifting on larger swarms.

V. Conclusion

In summary, we have explored the emergent behaviors of three related swarming rules derived from the original Icosystem swarm model. Assigning simple linear dynamics to these rules and using well-understood concepts from linear analysis, the following three possible behaviors emerge under different circumstances.

1) Clustering is the tendency of the swarm agents to simultaneously move toward a single point. Clustering can occur under any of the three rules. However, it is increasingly unlikely for "refugee" swarms as the number of agents grows. Clustering always occurs for connected "aggressor" swarms and strongly connected "protector" swarms.
2) Drifting, a situation in which all agents move in the same direction at the same (constant) speed, may occur only for "refugee" swarms.

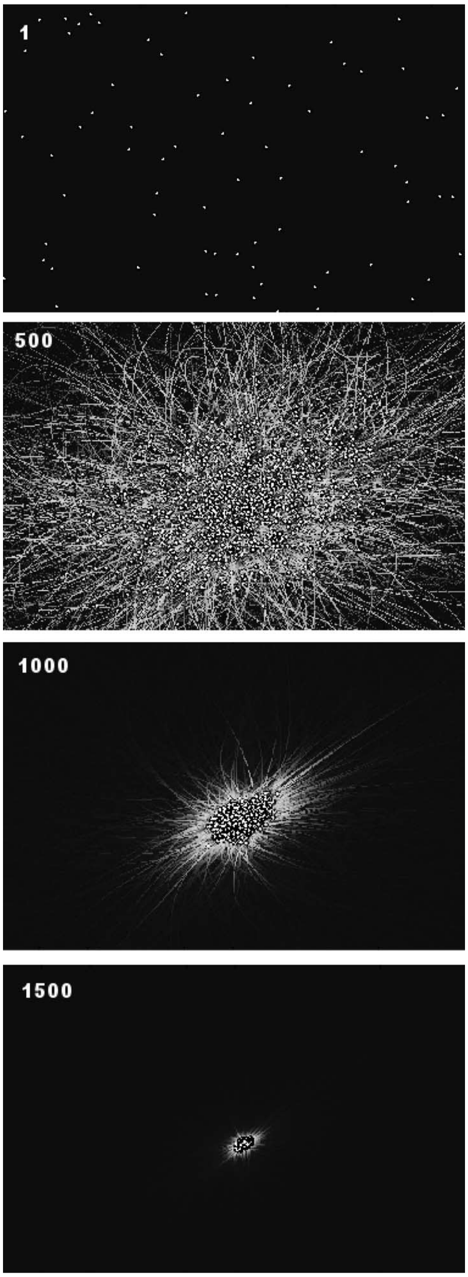

Fig. 5. One thousand randomly placed "protector" agents (some off screen) for $\alpha = 1/2$. Here, and in subsequent figures, the trajectory history of each agent is shown. All fields are 500×500 units. Snapshots of the convergence are shown for 1, 500, 1000, and 1500 steps using $\gamma = 0.01$. The agents, as predicted by the stability analysis, are converging to a point.

3) Explosion is an unstable behavior that occurs when agents diverge from each other. Strongly connected "protector" swarms and connected "aggressor" swarms cannot explode.

We note in closing that simulations of Icosytem-type swarms sometimes produce a behavior we term subgrouping, a condition in which several smaller swarms emerge, each with its own behavior. In light of the preceding analysis, subgrouping will arise when

$$\dim\{\text{null}(A)\} > 1.$$

A necessary (not sufficient) condition for subgrouping, then, is for a "protector" swarm to be weakly connected, or of course for "protector" or "aggressor" swarms to be nonconnected. In these cases, each subswarm will exhibit one of the three emergent behaviors listed above.

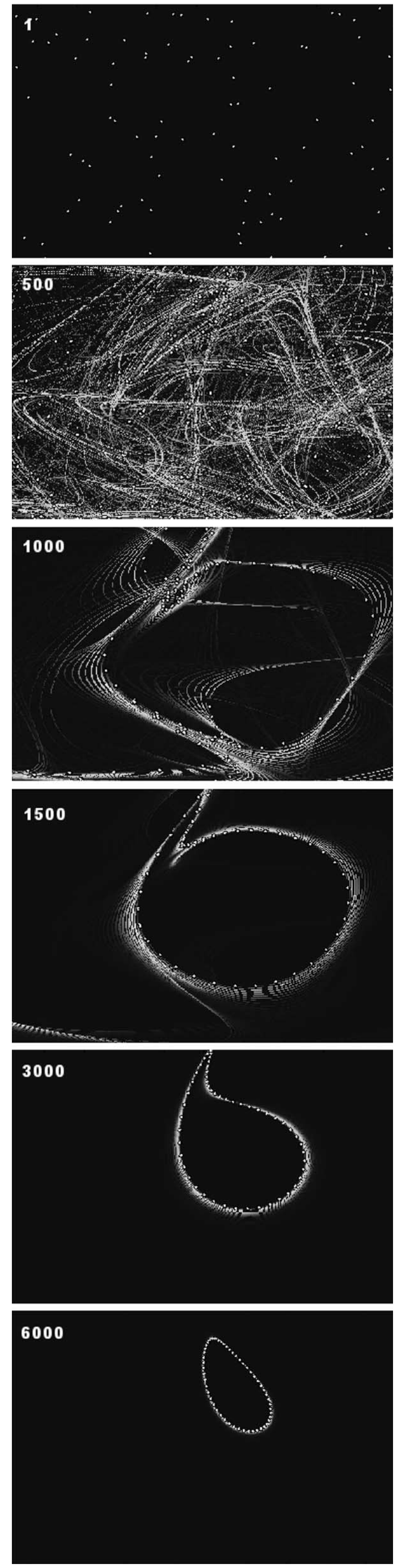

Fig. 6. For "aggressor" swarms, agents eventually cluster. "Follow-the-leader" behavior results in an ever-tightening loop.

This analysis is certainly not the last word on Icosystem-type swarms. It is entirely possible to formulate nonlinear dynamics that instantiate these rules, in which case questions of emergent behavior

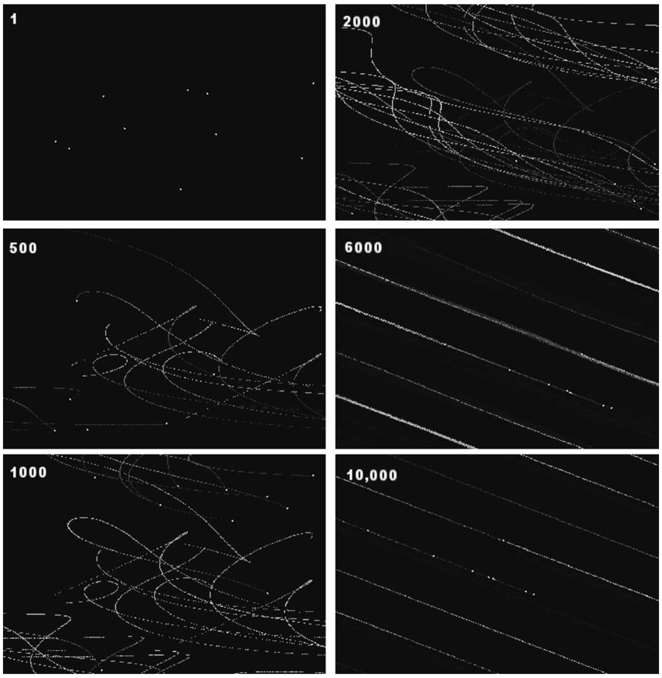

Fig. 7. Drifting behavior for "refugees." To keep view of the swarm, when an agent exits the right of the image, it immediately reenters the left, and similarly for the top and bottom. The swarm eventually moves in a straight line at constant velocity.

become much more difficult. One may instantiate simple nonlinear dynamics for the types of swarms studied here by updating the position of each agent sequentially (as opposed to simultaneously) or limiting the step size of an agent's movement to a constant (i.e., constant speed). These are illustrated and animated at the universal resource locator [21], and produce surprising behaviors. In fact, the original Icosystem Internet application [14] approximates linear dynamics with constant step-size increments for each agent. The qualitative behaviors described above still occur, with the exception that clustering swarms never appear to actually converge. (The constant step position updates prevent this and give the appearance of random motion near the cluster point.) Though certain stability arguments may still be possible, other behaviors such as chaos may also appear, while certain linear behaviors outlined in this correspondence may not occur at all.

Qualitative classification of emergent swarm behavior, independent of the specific dynamics that instantiate the swarm rules, may prove to be a very difficult problem in general. Even assuming such classification is possible generally, this correspondence reveals a certain "luck of the draw" aspect to behavior predication; for example, a given swarm may behave differently from one simulation to another based merely on the initial configuration of the agents and/or their topological connections. For example, if initial position $\mathbf{x}_0$ is chosen orthogonal to the m eigenvectors that correspond to zero eigenvalues, drifting behavior cannot occur. (The coefficients $w_1 \cdots w_m$ in (9) will be zero.) Simply knowing the rules is evidently insufficient to predict group behavior. Even in this correspondence, one can see a faint connection between syntactical, topological (e.g., graphical) and numerical/analytical methods that all come into play in the overall dissection of the problem. Such connections will need significantly more development in order to arrive at sufficiently abstract swarming models from which emergent behaviors can be deduced directly from syntactical rules.

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