STOCHASTIC RESONANCE OF A THRESHOLD DETECTOR: IMAGE VISUALIZATION AND EXPLANATION

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ABSTRACT

Stochastic resonance is said to occur when just the right amount of noise enhances the performance of a process. For a simple threshold detector, the first moment of stochastic resonance is obtained by passing the signal through a transfer function equal to a transposed and shifted version of the underlying noise's probability distribution function. The process is readily evident in images wherein noise corresponding to a linear transfer function produces a better visual representation than when other noise is used.

1. INTRODUCTION

Stochastic resonance is loosely defined as the phenomena in a detection process wherein the addition of just the right amount and type of noise improves performance. Too little or too much noise result in degraded performance. The phenomenon has numerous manifestations [1-9]. Pictures of the type illustrated in Figure 1 are used to illustrate a simple type of stochastic resonance when a simple threshold detector is used [9]. A gray level image is subjected to noise and is then subjected to a threshold. Too little noise renders the picture unrecognizable as does too much noise. When just the right amount of noise is added, a semblance of the original image is evident. This is even more evident in Figure 2 in which multiple realizations of the stochastic resonance phenomenon are averaged together to form a composite image that converges to the original image. In both Figures 1 and 2, the second image from the right is at a stochastic resonance.

2. STOCHASTIC RESONANCE OF A THRESHOLD DETECTOR

Although the effect in Figure 1 has been described as the "subjective response of our nonlinear perceptual system" [4], the phenomenon is readily explainable. Our derivation is for a continuous time signal. Its extension to images is straightforward.

Let the signal to be detected by the threshold detector be x(t). Distribution stationary noise, $\xi(t)$, is added and the sum is subjected to a threshold T to form the binary stochastic process

Figure 1: A simple example of stochastic resonance. The picture on the left is subjected to additive noise and is then thresholded. No noise (second from the left) and too much noise (on the right) render the image unrecognizable. At an intermediate resonant noise level, though, coarse structure is recognizable.

Figure 2: Time-averaged results of resonant phenomenon. Neither of the first two cases have enough noise added to them (although the second is closer). The third case corresponds to the "just right" level of noise. The fourth case, as above, corresponds to too much additive noise.

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$$Z(t) = U\big(x(t) + \xi(t) - T\big) \qquad (1)$$

where $U(\cdot)$, the unit step, is one for positive argument and is otherwise zero. The expected value of the process is

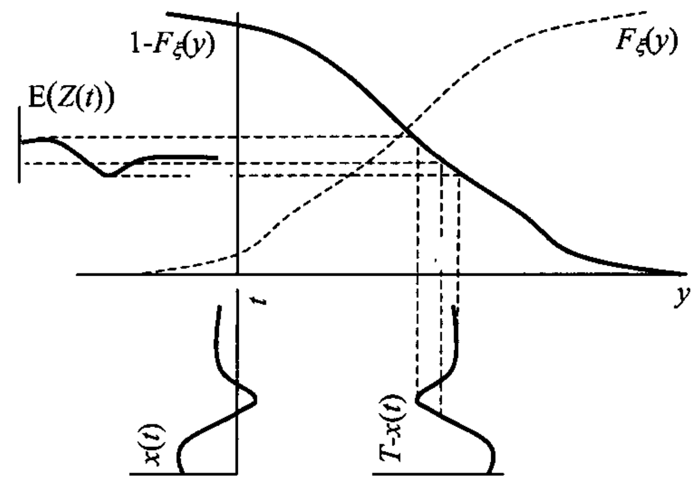

Figure 3. When noise, $\xi(t)$, is added to a signal, $x(t)$, the expected value of the thresholded sum can be visualized as here. The signal provides an input to a nonlinear transfer function determined by the probability distribution function of the additive noise.

$$\begin{aligned} \mathrm{E}\big(Z(t)\big) &= \mathrm{Prob}\big(x(t) + \xi(t) - T > 0\big) \\ &= \mathrm{Prob}\big(\xi(\mathrm{t}) > T\text{-}x(t)\big) \\ &= 1 - F_\xi\big(T - x(t)\big) \end{aligned} \qquad (2)$$

where $F_\xi(y) = \Pr\big(\xi \le y\big)$ is the cumulative probability distribution of the noise which, due to the assumption of the noise's distribution stationarity, is the same for all values of t. An illustration interpreting (2) is shown in Figure 3. A signal, $x(t)$, inverted and biased to form the signal T-$x(t)$, acts as the input a nonlinear transfer function, formed by the cumulative probability density function, to generate $\mathrm{E}(Z(t))$.

3. Stochastic Resonance In Uniform Noise

The process in Figure 3 can be used to explain the images in Figure 1. Assume the (one byte) image in the original image scales to [0,1]. The probability density function (pdf *a.k.a.* normalized histogram), $h_x(x)$, of such an image is then as illustrated in the bottom left of Figure 4. The image, x, is subjected to uniform white noise with probability density

Figure 4

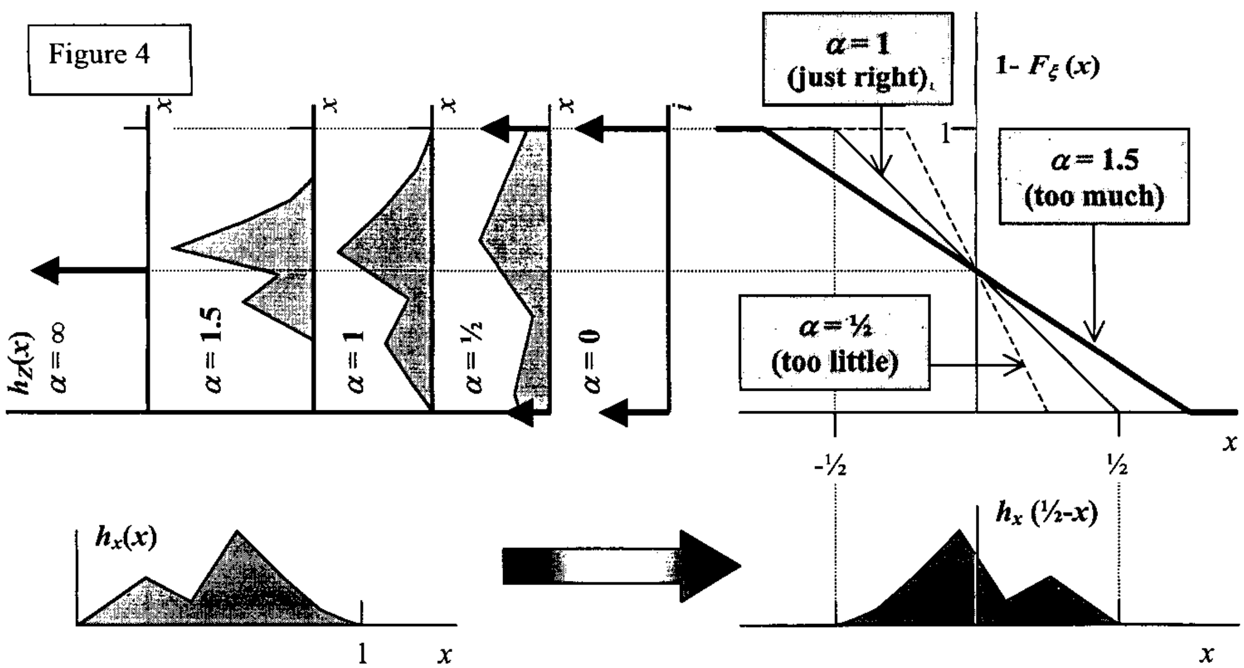

Figure 5: Multiple realizations of the stochastic resonance phenomenon averaged together. The images show a progression of 2, 5, 10, and 50 iterations of the averaging technique, to demonstrate the actual convergence to the original image due to the use of uniform random noise.

function $f_{\xi}(x) = \frac{1}{\alpha}\Pi\left(\frac{x}{\alpha}\right)$ where $\alpha \geq 0$ and $\Pi(\aleph)$ is equal to one for $|\aleph| \leq ½$ and is otherwise zero. As before, the noise, ξ, is added to the image, x, and subjected to a threshold to form a binary (0 or 1) image, Z. We choose a threshold of $T = ½$. From Figure 4, a parameter of $\alpha = 1$ produces an image Z such that $E[Z] = x$ exactly. Figure 5 demonstrates this phenomenon by showing the progression of convergence towards the original image by averaging an increasing number of realizations of the stochastically resonant images with the parameters described above. In general, the normalized histogram for $E[Z]$ is

$$h_Z(x) = k_X(x) + \beta_+\delta(x-1) + \beta_-\delta(x) \qquad (3)$$

where

$$k_X(x) = \alpha\, h_X\left(\frac{1}{2} - \alpha\left(x - \frac{1}{2}\right)\right),$$

$$\beta_+ = \int^{\infty} k_X(x)dx,\ \beta_- = \int_{-\infty}^{0} k_X(x)dx$$

and $\delta(\cdot)$ is the Dirac delta (impulse) function. An unscaled and undistorted version occurs in Equation (3) for $\alpha = 1.0$. From left to right in Figure 1, we have the original image, $\alpha = 0.0$, 1.0 and 1.5. As predicted in the analysis, "resonance" occurs at $\alpha = 1.0$.

Observing versions of the image subjected to different realizations of the noise in a rapidly framed movie allows the eye to average. Since the average converges to the expected value, the effect is more striking. Examples are available on the web [7,10].

Lastly [11], we note that, since $Z(t)$ is binary, $Z^2(t) = Z(t)$ and

$$\mathrm{var}\left(Z(t)\right) = \overline{Z(t)} - \left(\overline{Z(t)}\right)^2$$
$$= F_{\xi}(T - x(t))[1 - F_{\xi}(T - x(t))]$$

For the case of images, there is therefore certainty (*i.e.* zero variance) at 0 and 1. The uncertainty increases as we deviate from these values into midrange gray levels.

11. REFERENCES

1. B. Andò, S. Baglio, S. Graziani & N. Pitrone, "Threshold Error Reduction in Linear Measurement Devices by Adding Noise Signal", Proceedings of the IEEE Instrumentation and Measurements Technology Conference, St. Paul, Minnesota, May 18-21, 1998, pp.42-46.
2. F. Chapeau-Blondeau, "Noise-assisted propagation over a nonlinear line of threshold elements", Electronics Letters, Volume: 35 13 , 24 June 1999 , Page(s): 1055 –1056.
3. V. Galdi, V. Pierro & I.M. Pinto, "Evaluation of Stochastic Resonance Based Detectors of Weak Harmonic Signals in Additive White Gaussian Noise", Physical Review E, Vol.55, No.6,, 1998, pp.57-57.
4. S. Mitaim & B. Kosko, "Adaptive stochastic resonance" Proceedings of the IEEE , Nov. 98, v.86, no.11 (1998).
5. S. Mizutani, T. Sano & K. Shimohara, "Resonance in Chaotic Neural Networks Driven by a Weak Sinusoid", 1998.
6. Restrepo, A.; Zuluaga, L.F.; Pino, L.E, "Optimal noise levels for stochastic resonance", Acoustics, Speech, and Signal Processing, 1997. ICASSP-97, 1997 IEEE International Conference on Volume: 3 , 1997 , Page(s): 2365 -2368 vol.3
7. Enrico Simonotto, "Visual Perception of Stochastic Resonance", http://neurodyn.umsl.edu/sr/
8. S. Zozor & P.O. Amblard, "Stochastic Resonance in Discrete Time Nonlinear AR(1) Models, IEEE Transactions on Signal Processing, Vol.47, No.1, January 1999, pp.108-122.
9. F. Chapeau-Blondeau, "Stochastic Resonance in the Heaviside Nonlinearity with White Noise and Arbitrary Periodic Signal", Physical Review E, Volume 53, May 1996 , Pages: 5469-5472.
10. http://cialab.ee.washington.edu/Presentations/StochasticResonance/sr.html
11. F. Chapeau-Blondeau, "Input-Output Gains for Signal in Noise in Stochastic Resonance", Physics Letters A, Volume 232, 21 July 1997 , Pages: 41-48.