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Holographic representations of space-variant systems using phase-coded reference beams

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A new holographic implementation of a sampling technique permits, in principle, a straightforward representation of 2-D space-variant optical systems. The set of sample transfer functions required for the representation is recorded on a single holographic plate by utilizing phase coded reference beams. Because this approach does not depend on volume effects in the recording medium in an essential way, the holograms can be produced digitally, as well as optically. Basic concepts and preliminary experimental investigations related to this approach are presented and discussed.

Introduction

A procedure has recently been described for using a sampling technique to represent space variant optical systems.[1] One proposed implementation of this technique relies on volume hologram effects to angle-multiplex various sample transfer functions of the system into a thick recording medium.[2–4] A problem arises with angle multiplexing, however, in that the extinction angle effect is, for practical purposes, 1-D in nature due to the resulting Bragg cones.[3,5] Thus, using volume holograms, only 1-D space-variant systems can be represented in a straightforward way, without undesirable cross talk upon playback. More straightforward techniques have been demonstrated elsewhere for 1-D space-variant system representation.[6,7]

This paper demonstrates a different holographic implementation technique for sampling theorem-based representations of 2-D space-variant systems. This implementation utilizes reference beam encoding of the various transfer functions via a random (or pseudorandom) phase diffuser. This approach eliminates, in principle, the need for a volume recording medium, so that thin (rather than thick) recording media may be used. It therefore opens the important possibility of simulating arbitrary 2-D space-variant operations using computer-generated holograms not possible with volume hologram representations.

Phase encoded reference beams have been used previously for color holography[8,9] and for the purpose of multiplexing a number of point source objects for digital storage.[10] In the color holography application, simultaneous, but not coherent, readout of several different recordings (one for each primary color) is required. In the digital storage case, individual readout of each recorded object is desired with minimum background noise. The requirements in representing space-variant systems are more stringent in that simultaneous phase-coherent reconstruction of a number of recordings is desired with no significant objectionable cross talk between the various playback (reference) beams. As we will show, the random phase diffuser technique can satisfy these requirements.

Recording and Playback Operations with Phase-Coded Reference Beams

The space-variant system recording procedure based on the sampling theorem, but without reference beam phase encoding, is shown in Fig. 1.[2,3] The system S for the moment is shown sampled by impulse functions (point sources) i_1 to i_N, giving rise to the point spread functions h_1 to h_N at the output plane of S. The lens L_1 then produces fields proportional to the transfer functions H_1 to H_N at the hologram plane, where H_j is the Fourier transform of h_j.[11] Now for each input point source i_j there is a corresponding reference point source r_j which is transformed by the lens L_2 to form the reference plane wave R_j at the hologram plane.[2,3] The recording technique then consists of sequentially recording the interference patterns of the system function/reference plane wave pairs (H_j,R_j). If we bias the medium so that the resulting hologram's amplitude transmittance function t is proportional to the exposing intensity pattern,

$$t = \sum_{j=1}^{N} |H_j + R_j|^2. \tag{1}$$

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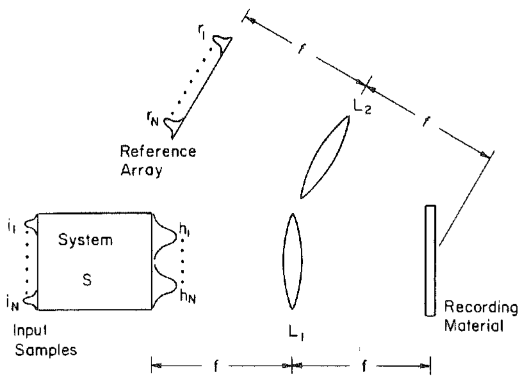

Fig. 1. Recoding scheme for the holographic representation of the space-variant system S. The holograms for the (i_j,r_j)pairs are recorded sequentially. Transforming lenses L_1 and L_2 each have focal length f.

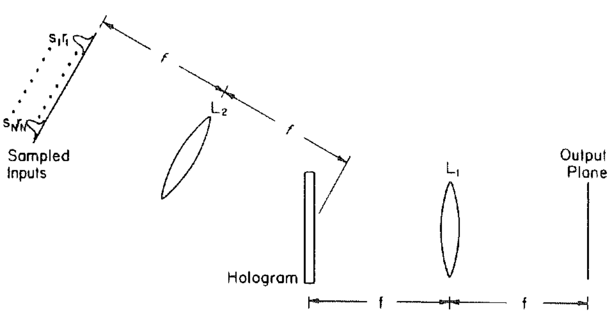

Fig. 2. Playback scheme for the holographic representation of the system S. The input is spatially sampled by a duplicate of the reference array.

The playback step is shown in Fig. 2. The input object is spatially sampled at the point source locations of the original reference array to produce sampled inputs s_1r_1 to s_Nr_N, where s_j is the sampled value at the jth location. The desired output plane field is given by the sum

$$o = \sum_{j=1}^{N} s_j \mathcal{F}^{-1}\{H_j\} = \sum_{j=1}^{N} s_j h_j, \tag{2}$$

where we note that coherent addition of the simultaneous reconstructions is required. Also note that potential cross-talk terms have been neglected,[12] and that low pass filtering in the hologram plane will be required to obtain a continuous output.[1]

Since cross-talk terms have been neglected in writing Eq. (2), we have assumed that each input point sample sees only the desired transfer function upon playback. We will now analyze what happens in the special case of a simple two sample point system to show what the cross-talk problems are and to examine how utilization of the diffuser encoding technique in the reference beams can suppress cross talk.

After sequential recording of the two object-reference pairs (H_1,R_1) and (H_2,R_2), the amplitude transmittance function stored in the hologram is given by

$$t = |H_1 + R_1|^2 + |H_2 + R_2|^2. \tag{3}$$

In the playback step, if the sampled input values are given by the constants s_1 and s_2, the reconstructed wavefront just to the right of the hologram in Fig. 2 is

$$\begin{aligned} W &= (s_1R_1 + s_2R_2)t \\ &= s_1R_1R_1^*H_1 + s_2R_2R_2^*H_2 + s_1H_2R_1R_2^* + s_2R_1^*R_2H_1 \\ &+ s_1H_1H_1^*R_1 + s_1R_1R_1^*R_1 + s_1H_2H_2^*R_1 + s_1R_2R_2^*R_1 \\ &+ s_2H_1H_1^*R_2 + s_2R_1R_1^*R_2 + s_2R_2R_2^*R_2 + s_2H_2H_2^*R_2 \\ &+ s_1R_1R_1H_1^* + s_1R_1R_2H_2^* + s_2R_1R_2H_1^* + s_2R_2R_2H_2^*. \end{aligned} \tag{4}$$

Here, the superscript * denotes a complex conjugate. Of the sixteen terms in Eq. (4), the first four have been diffracted by the hologram (see Fig. 2) and appear in the output plane. We will define these four terms as W', so that

$$W' \triangleq s_1R_1R_1^*H_1 + s_2R_2R_2^*H_2 + s_1R_1R_2^*H_2 + s_2R_1^*R_2H_1. \tag{5}$$

Since s_1, s_2, $R_1R_1^*$, $R_2R_2^*$, $H_1H_1^*$, and $H_2H_2^*$ are real numbers, we see that the second and third groups of four terms (each) in Eq. (4) are undiffracted terms which will not appear in the output plane of Fig. 2. Finally, the last four terms in Eq. (4) represent light which is diffracted to the opposite side of the playback system axis (analogous to a twin image effect) when compared with the first four terms which potentially appear in the output plane.[11] Thus we need only concern ourselves with the four terms listed in Eq. (5) when deciding which terms, after Fourier transformation by lens L_1 of Fig. 2, will appear in the output plane.

Now suppose an ideal phase diffuser is placed in the reference beam side of Figs. 1 and 2, such that r_1 and r_2 are unit amplitude waves with complex phase fronts (i.e., $r_1 = \exp[j\phi_1(x,y)]$, $r_2 = \exp[j\phi_2(x,y)]$). We may take the inverse Fourier transform[12] of Eq. (5) to show that the output (image) plane field is given by

$$\begin{aligned} o' = \mathcal{F}^{-1}\{W'\} &= s_1h_1 * (r_1 \star r_1) + s_2h_2 * (r_2 \star r_2) \\ &+ s_1h_2 * (r_1 \star r_2) + s_2h_1 * (r_2 \star r_1), \end{aligned} \tag{6}$$

where * represents convolution, and ☆ represents correlation. In obtaining Eq. (6) we have made use of the convolution and autocorrelation theorems of Fourier analysis.[11] Note that now, however, r_1 and r_2 represent the diffuser patterns seen at the two positions in the input sampling array. If the diffuser has the property that $r_i \star r_i$ is effectively a Dirac delta function, while $r_i \star r_j$ is a very broad, uniform spatial function, the output may effectively be expressed as

$$o' = s_1h_1 + s_2h_2 + \text{diffuse background noise}. \tag{7}$$

It should be noted that no extinction angle was assumed operative in obtaining the result of Eq. (7). In this approach, we are thus not restricted to working with thick recording media,[2,3] a potentially significant advantage. If, on the other hand, a volume hologram is used, the extinction angle effect can further reduce the average noise by restricting the cross talk to be only between input points lying along the loci defined by the

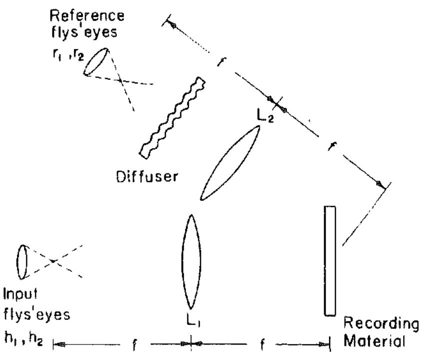

Fig. 3. Experimental setup for simulation of two-point magnifier. The elements of the fly's-eye arrays illuminated are vertical (in a line perpendicular to the plane of the drawing).

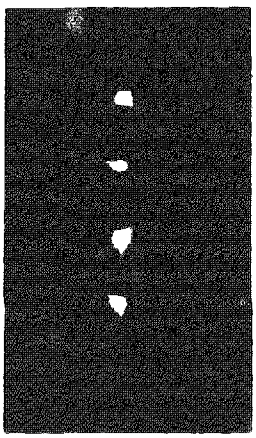

Fig. 4. Reconstructed output obtained with simulated two-point demagnifier and without diffuser-induced phase coding of reference beams. The two inner points are the desired output, while the two outer points represent cross talk.

relevant Bragg cones. Thus the existence of an extinction angle defined by the volume hologram, in addition to the cross talk-suppressing action associated with the diffuser, can effectively constrain cross-talk effects to N points at a time (assuming an N by N input sampling array) instead of the N^2 interactions expected if one uses only a diffuser and a thin recording medium.

Experimental Results

To test the theory presented above, an idealized, space-variant two-point magnifier was set up as shown in Fig. 3. Two fly's-eye lenses (3.8-mm diam each) having a vertical separation (in a line perpendicular to the plane of the figure) of 15.4 mm were used to form the reference point sources. The output of an idealized magnifier (with $M = \frac{1}{2}$) was simulated by using two fly's-eye lenses separated by 7.7 mm to form the object points. The transforming lenses L_1 and L_2 had 10-cm focal lengths. The results of four experiments which were performed are described below.

Experiment 1: In the first experiment no diffuser was placed in the reference beams. The (R_1,H_1) and (R_2,H_2) combinations were recorded sequentially on a Kodak 649F plate. Reconstruction was performed by moving L_1 10 cm to the right of the hologram and observing the image plane, which is now the Fourier transform plane of L_1 (see Fig. 2). The result is shown in Fig. 4 where the inner pair of points is the desired output (i.e., a pair of points separated by 7.7 mm), and the outer pair of points represents the cross-talk terms $\mathcal{F}^{-1}(s_1H_2R_1R_2{}^*)$ and $\mathcal{F}^{-1}(s_2H_1R_1{}^*R_2)$ which we want to suppress.

It should be noted here that the geometry of this experiment was such that the extinction angle effect was not operative in the vertical direction, so that the points in the vertical exhibit maximum cross talk, as shown in Fig. 4.

Experiment 2: In this experiment, a shower glass diffuser was inserted in the reference beams, about 3 cm from the plane of the reference point sources so that each of the two beams would intercept a maximum diffuser area, but would not intersect the diffuser area intercepted by the other reference beam. The output plane result for this experiment is shown in Fig. 5. Here the two desired output points stand out clearly against a diffuse background of coherent noise. The design pattern induced by the particular shower glass diffuser used did not have a very sharp autocorrelation function, so that repositioning of the developed hologram was not too critical an operation (i.e., it took a movement of several millimeters to make the output points disappear). Note that the diffuser also had a spatially varying attenuation, so that the cross talk is not spread as uniformly as possible over the output field. Nevertheless the results appear to be encouraging.

Experiment 3: A third experiment was performed to demonstrate the 2-D capabilities of the diffuser encoding technique. A three-point demagnifier was simulated with the same basic configuration as in Fig. 3. However, three of the four lenses in a square subarray portion of the fly's-eye array were used. The result, when the diffuser was placed in the reference

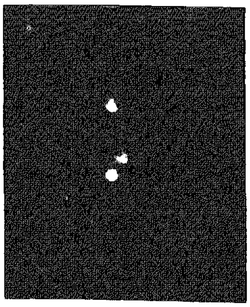

Fig. 5. Reconstructed output (Exp. 2) with phase coding of reference beams by shower glass diffuser. The cross-talk terms present in Fig. 4 have been spread out into diffuse background noise.

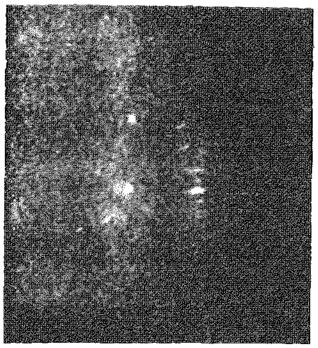

Fig. 6. Reconstructed output (Exp. 3) for the simulated three-point demagnifier, indicating diffuse background noise in both directions.

beams, is shown in Fig. 6. It can be seen that the multiple cross-talk points which would be expected without the diffuser encoding of the reference beams have been relegated to a diffuse background noise, while the three desired output points stand out clearly. Again it should be noted that since 649F has a rather large extinction angle relative to the angular separation of the reference beams used here, volume hologram effects were not responsible for eliminating either the horizontal or vertical cross talk expected without the diffuser.

Experiment 4: An experiment was performed to verify that even with the shower glass diffuser present in the path of the reference beams, coherent addition is obtained in the output plane as required based on Eq. (2). Basically the same setup as shown in Fig. 3 was used, with the exception that a single input point source was used in the object path (i.e., only h_1 present), whereas both reference fly's eyes were used. Unfortunately, due to the complicated nature of the shower glass diffuser pattern, it was difficult to document that indeed coherent addition of the outputs corresponding to the two reference beam point source inputs was taking place in the playback step. Indications are, however, that coherent addition was taking place, and additional experiments are underway to provide documentation of this hypothesis.

Conclusions

A technique has been demonstrated for holographically recording representations of 2-D space-variant systems using phase-encoded reference beams. The use of reference beam phase encoding for the purpose of multiplexing noninterfering holograms into a single thin recording medium, rather than attempting to use the extinction angle property of a thick recording medium to angle multiplex the multiple holograms, possesses a number of potential advantages. One would not be restricted to volume holograms for recording space-variant optical systems. Thus one could in principle construct computer-generated holograms for representing arbitrary 2-D space-variant processors.

It is clear that the correlation properties of the diffuser are important in determining the practical limitations of this space-variant recording process. Since a spatial sampling technique is used, one wants the diffuser autocorrelation function to be as nearly like a Dirac delta function as possible so that a high sampling rate can be achieved. This will in turn lead to precise positioning requirements for placement of the diffuser in the reconstruction step, suggesting *in situ* processing wherever possible. We also note that the cross correlation between any two different diffuser elements should be as broad as possible. This feature will optimally spread out the cross-talk noise spatially, so that one can multiplex as many samples as possible with (hopefully) a reasonable SNR on reconstruction. Additional investigations are underway in these and other areas relating to the technique presented in this paper.

As a final note, the scheme presented here is not limited to implementation of the sampling theorem. With minor changes in technique, other linear system representation models such as the piecewise isoplanatic approximation[4] and the orthonormal element response characterization method[13] can be similarly implemented.

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Appendix: Representations Using Volume Holograms

If the reference point sources represented by r_1 and r_2 are sufficiently far apart so that the Bragg extinction angle property of a volume recording medium is operative, the third and fourth terms in Eq. (5) will not appear in the output plane, since those terms represent cross talk between the two reference waves.[1,2] In this case Eq. (5) effectively becomes

$$W' = s_1R_1R_1{}^*H_1 + s_2R_2R_2{}^*H_2.$$

Assuming, for illustration purposes, that R_1 and R_2 have unit amplitudes, the output plane field is then given by

$$\begin{aligned} o' = \mathcal{F}^{-1}\{W'\} &= s_1\mathcal{F}^{-1}\{H_1\} + s_2\mathcal{F}^{-1}\{H_2\} \\ &= s_1h_1 + s_2h_2, \end{aligned}$$

which, based on Eq. (2), is the desired result. If, on the other hand, the Bragg extinction angle affect is not operative (i.e., the two points r_1 and r_2 are too close together), the inverse transform of the cross-talk terms $s_1H_2R_1R_2{}^*$ and $s_2H_1R_1R_2{}^*$ present in Eq. (5) will appear in the output plane. As explained in the main text, however, the use of phase-encoded reference beams can greatly suppress cross-talk effects and of course has the advantage of permitting straightforward representation of 2-D inputs.

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