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Correlation-Based Associative Memory and Its MOS Implementation

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Abstract. We propose an associative memory based on minimization of a free-energy function determined by the library vectors to be stored. When the library vectors are bipolar, the energy function contains minima at the library vector location. The minima can be sought by search techniques such as gradient descent. Significantly, if the correlation nonlinearity is chosen to be sufficiently strong, then convergence occurs in a single step. We demonstrate how the algorithm can be implemented using MOS circuitry.

1. Introduction

Associative memory architectures have been presented that are based on artificial neural network structures [1, 2] and iterative matched filters. Indeed, in certain configurations, the resulting procedures produce numerically identical results [3].

Similar associative memories based on nonlinear operations in the correlation domain have also been proposed [4-9]. Such memories have shift-invariant characteristics [6] and high capacity [10, 11]. If the correlation nonlinearity is stronger than $z(x) = N^{x/2}$ and the library vectors are bipolar (i.e., contain only ±1's), then the iterations normally required in such memories need not be used. Convergence, rather, occurs in one step [7-9].

In this paper, we examine the correlation based associative memory and show its relation to minimization of a free-energy function placed on the training data. Using gradient descent, the search is shown to converge in a single step for the case of a bipolar library.

A circuit implementation of the resulting associative memory is proposed. The correlation operation is performed using metal oxide semiconductor (MOS) capacitors and MOS switches. The subthreshold region of MOS is used to implement the exponential nonlinearity.

2. Probability of Occurrence and Free Energy

Let $\{\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_N\}$ be library vectors of dimension L that we wish to store in the associative memory. Let $\mathbf{f}$ be a vector of like dimension. Then, the probability $P(\mathbf{f})$ is

$$P(\mathbf{f}) = \sum_{i=1}^{N} P(\mathbf{f}_i)P(\mathbf{f}\backslash\mathbf{f}_i) = \sum_{i=1}^{N} Q_i P(\mathbf{f}\backslash\mathbf{f}_i)$$

where $\mathbf{f}$ is an arbitrary vector and Q_i is the priori probability of the library vector $\mathbf{f}_i$. We define an energy function $F(\mathbf{f})$ as

$$F(\mathbf{f}) = -\gamma \ln[P(\mathbf{f})] = -\gamma \ln \left[\sum_{i=1}^{N} Q_i P(\mathbf{f}\backslash\mathbf{f}_i)\right]$$

where γ is a constant. Note that $F(\cdot)$ is analogous in concept to free energy [12]. Our associative memory paradigm is as follows. Let $\mathbf{g}$ denote a distorted version of one of the library vectors. With an initialization of $\mathbf{g}$, we search the landscape of F for a minimum. We will show that the minima occur at library vectors location in many important cases.

Example 2.1. If $P(\mathbf{g}\backslash\mathbf{f}_i)$ has an identically independent Gaussian distribution, then

$$P(\mathbf{g}\backslash\mathbf{f}_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\| \mathbf{g} - \mathbf{f}_i \|^2}{2\sigma^2}\right]$$

Let $\gamma = 2\sigma^2$ and $a = 1/\gamma$. Then

$$F(\mathbf{g}) = -\frac{1}{a} \ln \left[\sum_{i=1}^{N} Q_i \exp\{-a \| \mathbf{g} - \mathbf{f}_i \|^2\}\right] + C \quad (1)$$

where C is a constant. An example of the energy landscape is illustrated in figure 1 for $a = 8$ and $a = 32$ with $C = 0$. The library vectors are $L = 1$ dimensional and are $\{-1, 0, 0.5, 1\}$ for both values of a. The energy function reaches local minima at the neighborhood of

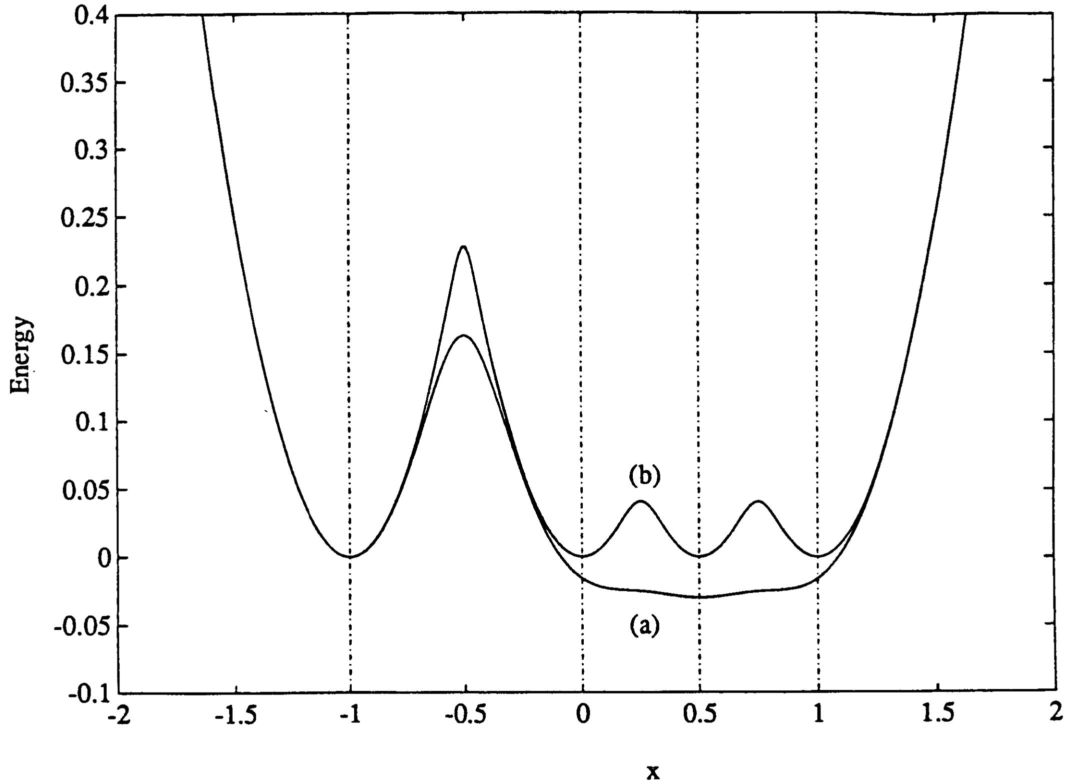

Fig. 1. The energy in the Gaussian noise for (a) $a = 8$ and (b) $a = 32$. The library vectors (scalars in this example) are −1, 0, 0.5, and 1.

the library vector location when $a = 32$, but this is not the case when $a = 8$. Thus we see that the minima become more distinct for large values of a (which, in the free-energy analogy, is inversely proportional to temperature).

Example 2.2. If all vectors are bipolar (i.e., contain only ± 1 components) and the noise is independent flip (Bernoulli) noise with probability $p < 1/2$, then, under the assumption that $\mathbf{f}$ is bipolar,

$$P(\mathbf{g} \backslash \mathbf{f}_i) = p^{k_i}(1 - p)^{L-k_i} \tag{2}$$

where k_i is the Hamming distance between $\mathbf{g}$ and $\mathbf{f}_i$. Equation (2) can be written as

$$P(\mathbf{g} \backslash \mathbf{f}_i) = (1 - p)^L \left(\frac{p}{1-p}\right)^{k_i} = (1 - p)^L \exp[-k_i b] \tag{3}$$

where

$$b = \ln\left(\frac{1-p}{p}\right)$$

and $\| \mathbf{g} - \mathbf{f} \|^2 = 4k_i$. Equation (3) can then be written as

$$P(\mathbf{g} \backslash \mathbf{f}_i) = (1 - p)^L \exp[-a \| \mathbf{g} - \mathbf{f}_i \|^2]$$

and let $a = b/4$. It follows that the energy function has the same form as (1)

$$F(\mathbf{g}) = -\gamma \ln \left[\sum_{i=1}^{N} Q_i \exp\{-a \| \mathbf{g} - \mathbf{f}_i \|^2\} \right] + C \tag{4}$$

3. Derivation of Operation

Searching for the maximum of $P(\mathbf{g})$ is equivalent to searching for the minimum of $F(\mathbf{g})$. The gradient descent method is a popularly used search technique and can be written as

$$\mathbf{g}(n + 1) = \mathbf{g}(n) - \beta \nabla_g F(\mathbf{g}) \tag{5}$$

where $\nabla_g F(\mathbf{g})$is gradient of $F(\mathbf{g})$. If initialization of the search is close to minimum, say $\mathbf{f}_1$, then we expect that $\mathbf{g}(\infty) = \mathbf{f}_1$.

3.1. Continuous Level Library Vector

In Example 2.1, the gradient is

$$\nabla_g F(\mathbf{g}) = -\frac{1}{a} 2a \frac{\sum_{i=1}^{N} Q_i(\mathbf{g} - \mathbf{f}_i) \exp(-a \parallel \mathbf{g} - \mathbf{f}_i \parallel^2)}{\sum_{i=1}^{N} Q_i \exp(-a \parallel \mathbf{g} - \mathbf{f}_i \parallel^2)}$$

$$= 2\mathbf{g} - 2\frac{\sum_{i=1}^{N} Q_i\mathbf{f}_i \exp(-a \parallel \mathbf{g} - \mathbf{f}_i \parallel^2)}{\sum_{i=1}^{N} Q_i \exp(-a \parallel \mathbf{g} - \mathbf{f}_i \parallel^2)}$$

Equation (5) is

$$\mathbf{g}(n+1) = \mathbf{g}(n) - \beta \nabla_g F(\mathbf{g})$$
$$= (1 - 2\beta)\mathbf{g}(n) + 2\beta \frac{\sum_{i=1}^{N} Q_i\mathbf{f}_i \exp(-a \parallel \mathbf{g} - \mathbf{f}_i \parallel^2)}{\sum_{i=1}^{N} Q_i \exp(-a \parallel \mathbf{g} - \mathbf{f}_i \parallel^2)}$$

Thus, if $\parallel \mathbf{f}_i \parallel$ is the same for all library vectors and $\beta = 1/2$, then

$$\mathbf{g}(n+1) = \frac{\sum_{i=1}^{N} Q_i\mathbf{f}_i \exp[2a\alpha_i(n)]}{\sum_{i=1}^{N} Q_i \exp(2a\alpha_i(n)]} \quad (6)$$

where $\alpha_i(n) = \mathbf{g}^T \mathbf{f}_i$ is the correlation of $\mathbf{g}$ with $\mathbf{f}_i$.

3.2. Bipolar Library Vector

For the bipolar library with equal priori probability, a sufficiently large value of a and corresponding value of β, assures that the gradient descent method converges in one step, if, after the iteration, we project onto the nearest bipolar vector. For the more general case where the nonlinearity is not sufficiently strong, we calculate from (4).

$$\nabla_g F(\mathbf{g}) = \gamma a \left\{ \mathbf{g} - \frac{\sum_{i=1}^{N} Q_i\mathbf{f}_i \exp(2a\alpha_i)}{\sum_{i=1}^{N} Q_i \exp(2a\alpha_i)} \right\}$$

where $\alpha_i = \mathbf{f}^T\mathbf{g}$. If we choose the gain

$$\beta = \frac{1}{\gamma a}$$

then the recurrent relation will be (6). Generally, $\mathbf{g}(n + 1)$ here is not bipolar, so we project it onto the nearest bipolar vector by the sign(•) operation

$$\mathbf{g}(n+1) = \text{sign}\left[\frac{\sum_{i=1}^{N} Q_i\mathbf{f}_i \exp[2a\alpha_i(n)]}{\sum_{i=1}^{N} Q_i \exp[2a\alpha_i(n)]} \right] \quad (7)$$

The block diagram of this operation is shown in figure 2, and this operation is the same as the exponential function at the correlation domain in iterative matched filter [7, 8].

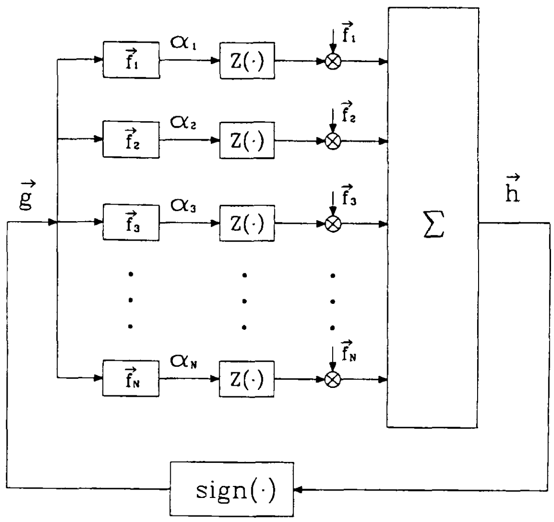

Fig. 2. Block diagram of a matched filter with a nonlinearity in the correlation domain. $A(x) = e^{2ax}$.

3.3. One Step Convergence for Bipolar Library

Consider the case where the priori probability, Q_i, is the same for all library vectors. One step convergence can then be assured for the iteration in (7). Specifically, if $a > \ln(N - 1)/2$, the operation in (7) converges to one of the library vector in one step [7, 8]. This condition is the same as requiring that $p < 1/N$. Under this condition, we establish the following

Lemma 3.1. Let $\mathbf{g}$ and $\mathbf{f}_i$ be bipolar vectors. If $a > \ln(N - 1)/2$, then $F(\mathbf{g}) > F(\mathbf{f}_i)$ for $\mathbf{g} \neq \mathbf{f}_i$ for all $i = 1, 2, \ldots, N$.

Proof. Let $c_{ik} = h(\mathbf{f}_i, \mathbf{f}_k)$ and $d_k = h(\mathbf{g}, \mathbf{f}_k)$ where $h(\mathbf{x}, \mathbf{y})$ is the Hamming distance between $\mathbf{x}$ and $\mathbf{y}$. Let

$$A(\mathbf{f}_i, \mathbf{g}) = \sum_{k=1}^{N} e^{-4ac_{ik}} - \sum_{k=1}^{N} e^{-4ad_k} \tag{8}$$

Because $h(\mathbf{f}_i, \mathbf{f}_k) \leq h(\mathbf{f}_i, \mathbf{g}) + h(\mathbf{f}_k, \mathbf{g})$,

$$e^{-4ac_{ik}} \geq e^{-4ad_i} e^{-4ad_k}$$

Equation (8) then becomes

$$A(\mathbf{f}_i, \mathbf{g}) \geq 1 + \sum_{k \neq i}^{N} [e^{-4ad_i} e^{-4ad_k}] - e^{-4ad_i} - \sum_{k \neq i}^{N} e^{-4ad_k}$$

$$= 1 - \sum_{k \neq i}^{N} e^{-4ad_k} - e^{-4ad_i} \left[1 - \sum_{k \neq i}^{N} e^{-4ad_k} \right]$$

$$= [1 - e^{-4ad_i}] \left[1 - \sum_{k \neq i}^{N} e^{-4ad_k} \right]$$

If $a > \ln(N - 1)/2$, then, for $n \geq 1$

$$e^{4an} \leq \frac{1}{N - 1}$$

This leads to conclude that $A(\mathbf{f}_i, \mathbf{g}) \geq 0$ and, as a consequence, $F(\mathbf{g}) > F(\mathbf{f}_i)$. □

The above lemma establishes that the free energy has local minima at all library vector locations. One step convergence using (7) attains this local minimum without iteration.

4. Circuit Implementation of the Operation

The operation in equation (7) consists of two parts. The first is calculation of the correlation between the input and each library element. The second is generation of the nonlinearity operation and sign(·) function operations. The implementation of correlation can be achieved a matrix vector multiplier and that of the nonlinear function by a cascade connection of the multiple differential amplifier and a high-gain amplifier.

We give an example of the implementation using CMOS technology. The bipolar matrix vector multiplier can be implemented by MOS switches and MOS capacitors. Figure 3 is an example for $\mathbf{f} = [1, -1, -1, 1]^T$. The connection between capacitors and the voltage sources depend on the values of elements of **f**. Also, the switches are operated by the values of input **g**. The operation is as follows

1. Charging mode:
   a. The switch S_5 is connected to V_{DD}.

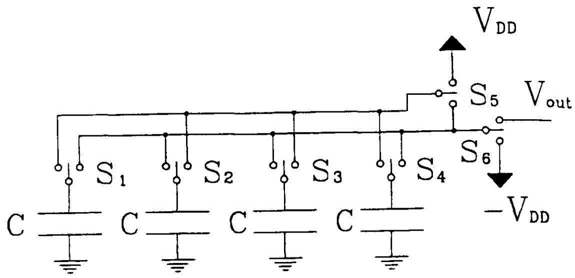

Fig. 3. An example of MOS implementation of correlation with a bipolar library vector.

   b. The switch S_6 is connected to $-V_{DD}$.
   c. The switches S_1, S_2, S_3, and S_4 are connected to the left side if the input is 1 and to the right side otherwise.
2. Distributing mode:
   a. The switches S_1, S_2, S_3, and S_4 are connected to the right side.
   b. The switch S_5 is closed to the lower terminal.
   c. The switch S_6 is closed to V_{out}.

We will explain the operation of the circuit for $\mathbf{g} = [1, 1, 1, -1]^T$. After the charging mode, the voltages of the capacitors are V_{DD}, $-V_{DD}$, $-V_{DD}$, $-V_{DD}$. In the distributing mode, because the switch 5 is connected to lower terminal, all capacitors are connected parallel. The voltage across each capacitor is the same and the value is $-2V_{DD}/4$. The value -2 is the same as $\mathbf{f}^T\mathbf{g}$, so this circuit gives the correlation operation.

Figure 4 shows the basic schematic circuit for nonlinearity implementation. If all of the MOS transistors are operated in the subthreshold region, then the current voltage relationship of the NMOS follows as [13–15].

$$I_D = I_{do} \exp\left(\frac{\kappa q V_g}{kT}\right) \left[\exp\left(-\frac{qV_s}{kT}\right) - \exp\left(-\frac{qV_d}{kT}\right)\right]$$

If $\kappa V_g - V_d$ is negative, then we can ignore the last term of the above equation. In figure 4,

$$I_1 = \sum_{i=1}^{p} I_{do_i}^{+} \exp\left[\frac{q(\kappa V_i^+ - V_s)}{kT}\right]$$

$$I_2 = \sum_{j=1}^{r} I_{do_j}^{-} \exp\left[\frac{q(\kappa V_j^- - V_s)}{kT}\right]$$

Using the Kirchhoff's law, $I_o = I_1 + I_2$. The current mirror action by T_1 and T_2 leads to $I_{out} = I_1 - I_2$. Then

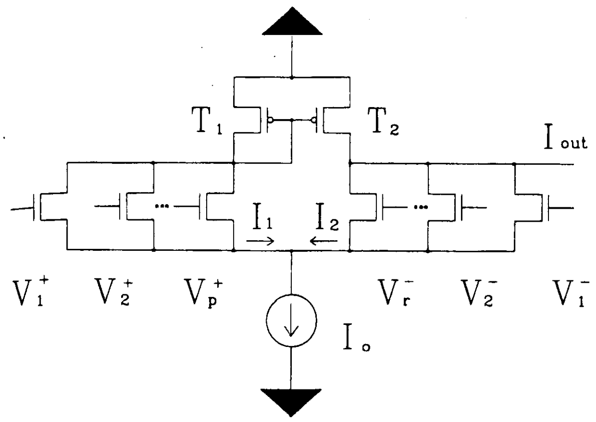

Fig. 4. A CMOS implementation of correlation with a bipolar library vector.

$$I_{\text{out}} = I_o \frac{\sum_{i=1}^{p} I_{do_i}^{+} \exp(uV_i^{+}) - \sum_{j=1}^{r} I_{do_j}^{-} \exp(uV_j^{-})}{\sum_{i=1}^{p} I_{do_i}^{+} \exp(uV_i^{+}) + \sum_{j=1}^{r} I_{do_j}^{-} \exp(uV_j^{-})}$$

where

$$u = \frac{\kappa q}{kT}$$

Here, the I_{do}'s are the function of the *W/L* value of gate.

Figure 5 is an example of the circuit implementation whose library vectors are

$$\mathbf{f}_1 = [1, -1, 1, -1, -1]^T$$
$$\mathbf{f}_2 = [1, -1, -1, -1, 1]^T$$
$$\mathbf{f}_3 = [1, -1, 1, 1, -1]^T$$

In this figure, the circuit has three stages. The first is the correlation operation using MOS switches and capacitors. The second is the nonlinear operation using the CMOS which is operated exponentially in the subthreshold region. The third is a saturated high gain amplifier, i.e., the comparator or buffer with switches.

5. Conclusion

In this paper, we found the most probable solution for correlation based associative memories using an energy function equal to the logarithm of the probability. For

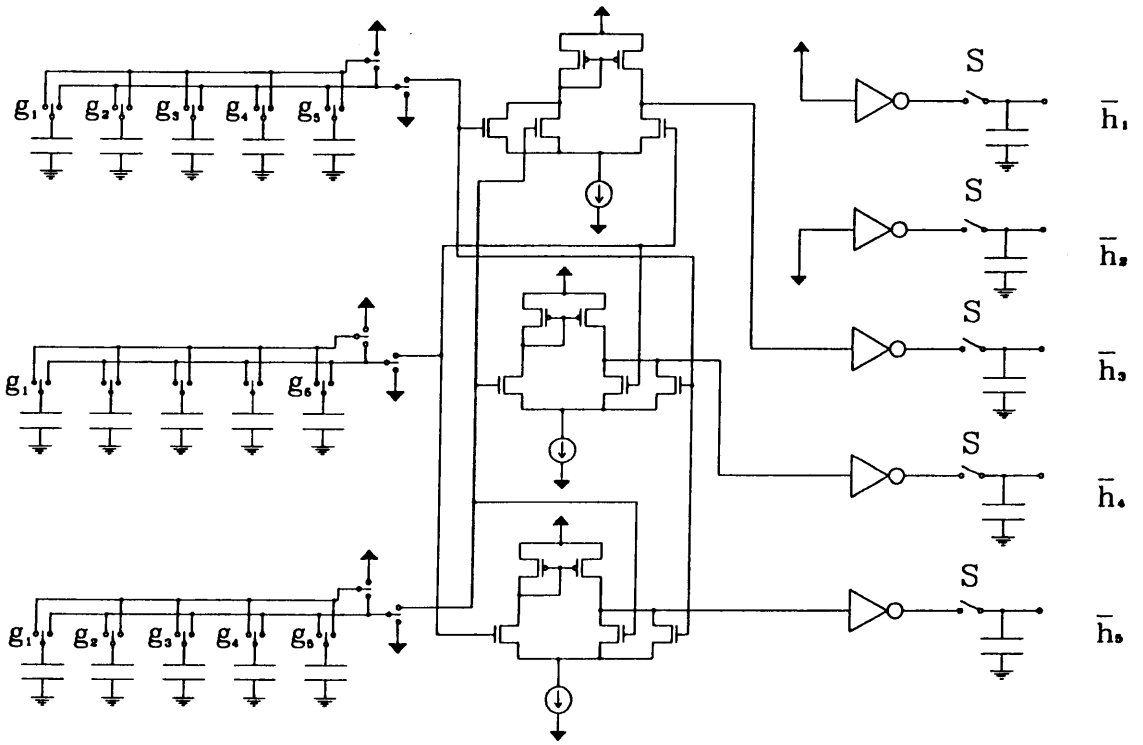

Fig. 5. The schematic of the CMOS implementation for the library $[1, -1, 1, -1, -1]^T$, $[1, -1, -1, -1, 1]^T$, and $[1, -1, 1, 1, -1]^T$. The switches in the correlation oprator operate by $g_1(n)$, $g_2(n)$, $g_3(n)$, $g_4(n)$, and $g_5(n)$. The outputs are $\bar{h}_1(n)$, $\bar{h}_2(n)$, $\bar{h}_3(n)$, $\bar{h}_4(n)$, and $\bar{h}_5(n)$ where $h_i(n) = g_i(n + 1)$. The switches S are closed in the distributing mode.

bipolar library vectors, the associative memory with an exponential nonlinearity in correlation domain minimizes the energy which is proportional to ln[P(f)]. Also, we showed that one-step convergence to the desired local minimum can occur for a bipolar library with equal priori probabilities for sufficiently strong nonlinearity. We demonstrated that this associative memory can be implemented using the CMOS technology.

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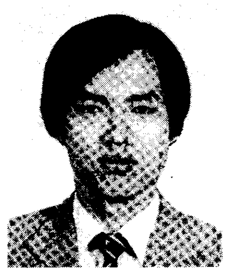

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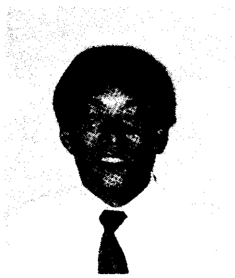

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