R.J. Marks II, L.E. Atlas, J.J. Choi, S. Oh, K.F. Cheung and D.C. Park ,
"Performance analysis of associative memories with nonlinearities in the correlation domain",
Applied Optics, vol. 27, pp.2900-2904 (1988).

Performance analysis of associative memories with nonlinearities in the correlation domain

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A matched filter-based architecture for associative memories (MFAMs) has been proposed by many researchers. The correlation from a leg of a matched filter bank, after being altered nonlinearly, weights its corresponding library vector. The weighted vectors are summed and clipped to give an estimate of the library vector closest to the input. We analyze the performance of such architectures for binary and/or bipolar inputs and libraries. Sufficient conditions are derived for the correlation nonlinearity so that the MFAM outputs the correct result. If, for example, N bipolar library vectors are stored, the correlation nonlinearity $Z(x) = N^{x/2}$ will always result in that library vector closest to the input in the Hamming sense.

I. Introduction

Consider the following classic detection theory problem. We have N library vectors of equal norm, $\{\mathbf{f}_n|1 \le n \le N\}$. Given a perturbed version of one of these library vectors $\mathbf{g}$, we wish to find the best match in the library. In many scenarios, the matched filter is optimal for performing this task.[1–3] The vector $\mathbf{g}$ is correlated with each library vector. The maximum correlation points to the chosen vector. The matched filter always yields the library vector closest to $\mathbf{g}$ in the mean square sense. Other performance criteria that can be achieved by the matched filter bank are maximization of SNR and minimization of the probability of error.

An associative memory, when fed an input $\mathbf{g}$, should produce as output the library vector closest to $\mathbf{g}$ in some sense. Thus, in the same scenarios where the matched filter is optimum, a matched filter bank followed by a search for the maximum correlation and a corresponding table lookup, is an optimal associative memory architecture.

The matched filter bank can be implemented in parallel. A search for the maximum correlation coefficient, however, is inherently serial. We may sacrifice optimality for parallelness by using the matched filter outputs to weight each corresponding library object.[4–7] The weighted terms are summed to give an estimate of the memory output. If the library is known to be binary, this output can be clipped to possibly improve the result. The output can then, in turn, be iteratively fed into the memory. Certain neural network associative memories[8–10] are algorithmically identical to this procedure.

In this paper, we show that for binary objects, matched filter architectures are optimum for finding that library object closest in the Hamming sense to the input. The use of nonlinearities in the correlation domain is also considered. Sufficient constraints for the matched filter associative memory (MFAM) to operate successfully are explored for a number of cases.

II. Matched Filter Associative Memory

In this section, we develop a generalized matched filter approach to associative memory architectures. Let $\mathcal{F} = \{\mathbf{f}_n|1 \le n \le N\}$ denote a set of bipolar (+1,−1) library vectors of length L and $\mathbf{g}$ a perturbed bipolar version of one of the library vectors. An associative memory architecture for finding that vector is shown in Fig. 1. The input vector is correlated with each library vector to form $\alpha_n = \mathbf{g}^T\mathbf{f}_n$ or, in vector form, $\alpha = \mathrm{F}^T\mathbf{g}$, where the library matrix is defined by $\mathrm{F} = [\mathbf{f}_1|\mathbf{f}_2|\mathbf{f}_3| \ldots |\mathbf{f}_N]$.

Each correlation coefficient is then operated on by a point nonlinearity $Z_n(\cdot)$, which in turn weights the corresponding library vector. The weighted vectors are summed to obtain $\mathbf{h} = \Sigma_{n=1}^N Z_n(\alpha_n)\mathbf{f}_n$. Each element of $\mathbf{h}$ is fed through a hard limiter $C[\cdot] = \mathrm{sgn}[\cdot]$. The output is thus

$$\mathbf{f}^* = C\left[\sum_{n=1}^{N} Z_n(\alpha_n)\mathbf{f}_n\right].$$

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Received 10 July 1987.
0003-6935/88/142900-05$02.00/0.

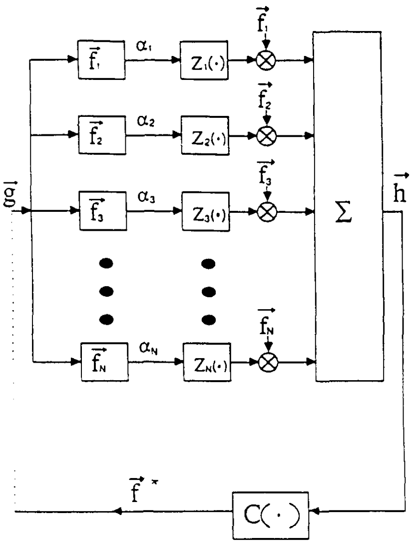

Fig. 1. Block diagram of a matched filter-based associative memory with nonlinearities in the correlation domain.

Ideally, **f** should be that library vector closest to **g** in some sense.

II. Extensions

1. The output $\mathbf{f}^*$ could be again fed into the processor to yield a possibly better result. We refer to such a processor as an iterative matched filter associative memory (IMFAM).
2. We will also want to examine the MFAM performance when the input or matched filter bank is binary (0,1). As such, define

$$\hat{\mathbf{f}}_n = \tfrac{1}{2}(\mathbf{f}_n + 1),$$

$$\hat{\mathbf{h}}_n = \tfrac{1}{2}(\mathbf{h}_n + 1),$$

where 1 is a vector of ones. Clearly, the hatted vectors are binary when the unhatted vectors are bipolar (−1,+1). For binary inputs, the hard limiter performs a unit step operation.

III. Special Cases

1. When operated iteratively with a binary input and bipolar matched filter, IMFAM performs identical (yet architecturally different) to the synchronous version of Hopfield's content addressable memory (CAM) with autoneural interconnects when $Z_n(x) = x$.[8,10]
2. Soffer *et al.*[5,12–13] implemented a nonlinear holographic associative memory with feedback using a hologram and phase-conjugate mirrors (PCMs). The hologram acts the memory element. The PCMs were used for feedback, thresholding, and amplification to improve the correlation between the input object and desired image.
3. Athale *et al.*[11] suggested library vector-dependent weights for attentive associative memory. Specifically, $Z_n(x) = a_n x$, where a_n is an attentive factor.
4. Psaltis *et al.*[4] used square law and unit step nonlinearities in a MFAM optical architecture.
5. We justify a form of MFAM from a receiver design point of view. Let **g** be a library vector perturbed by additive white Gaussian noise. The corresponding decision function is[1]

$$d(\mathbf{f}_n) = \frac{\pi_n}{(2\pi\sigma^2)^{L/2}} \exp - \left(\frac{\|\mathbf{g} - \mathbf{f}_n\|^2}{2\sigma^2}\right),$$

where π_n is *a priori* probability, i.e., $\pi_n = Pr\{\mathbf{f}_n\}$. Hence, if the decision function for $\mathbf{f}_n$ is comparatively large, it should dominate the expression

$$\mathbf{u} = \sum_{n=1}^{N} d(\mathbf{f}_n)\mathbf{f}_n. \tag{1}$$

If the library vectors are bipolar, then $\|\mathbf{f}_n\|^2 = L$, and we can write

$$\mathbf{u} = a \sum_{n=1}^{N} \pi_n \exp\left(\frac{\mathbf{f}_n^T\mathbf{g}}{\sigma^2}\right)\mathbf{f}_n,$$

where

$$a = \frac{1}{(2\pi\sigma^2)^{L/2}} \exp\left[\frac{(L + \|\mathbf{g}\|^2)}{2\sigma^2}\right]$$

is simply a proportionality constant. Thus **h** is proportional to **u** when we use

$$Z_n(x) = \pi_n \exp(x/\sigma^2). \tag{2}$$

If all prior probabilities are equal (i.e., $\pi_n = 1/N$), they can be absorbed into the constant.

IV. Hamming Distance and Correlation

As discussed on Sec. I, matched filters are used largely due to their optimal performance in certain scenarios. If **g** is a library vector perturbed by white Gaussian noise, choosing the library vector corresponding to the largest correlation coefficient minimizes the probability of error and maximizes the SNR.[1,2] Below we show four cases where matched filters also minimize the Hamming distance.

1. Let **g** be a bipolar library vector at a Hamming distance of h_n from the bipolar vector $\mathbf{f}_n$. Then

$$\alpha_n = L - 2h_n, \tag{3}$$

where L is the length of the library vectors. Thus maximizing correlation is the same as minimizing the Hamming distance.

2. In some situations (e.g., Hopfield's model) the binary vectors are stored at the memory as bipolar forms. To observe this binary input, bipolar matched filter case, the correlation coefficient is

$$\beta_n = \tfrac{1}{2}(\mathbf{g} + 1)^T\mathbf{f}_n$$
$$= \tfrac{1}{2}\alpha_n + \tfrac{1}{2}S_n,$$

where S_n is the sum of the elements in $\mathbf{f}_n$. Substituting Eq. (3) gives

$$\beta_n = \tfrac{1}{2}(L + S_n) - h_n. \tag{4}$$

If we assume

$$S_n = \sum_{m=1}^{L} f_{nm} = S$$

for all n, Eq. (4) becomes $\beta_n = \frac{1}{2}(L + S) - h_n$. (This constraint has a similar flavor to that of requiring each library vector to have the same energy. Such energy constraints are common in matched filter detectors.[1]) Then maximizing the binary-bipolar correlation is the same as minimizing the Hamming distance.

3. If both input and library vectors are binary, the correlation coefficients are

$$\gamma_n = \frac{1}{4}(\mathbf{g} + 1)^T(\mathbf{f}_n + 1)$$
$$= \frac{1}{2}L + \frac{1}{4}(S_g + S_n) - \frac{1}{2}h_n, \quad (5)$$

where S_g and S_n are the total number of ones in $\mathbf{g}$ and $\mathbf{f}_n$, respectively. Again, if $S_n = S$, Eq. (5) can be maximized by minimizing h_n.

4. Finally, the bipolar input-binary filter correlation coefficient is

$$\lambda_n = \frac{1}{2}\mathbf{g}^T(\mathbf{f}_n + 1)$$
$$= \frac{1}{2}(S_g + L) - h_n.$$

Thus we have a maximum correlation by choosing a minimum Hamming distance. The above four cases are summarized in Table I.

V. MFAM Performance

In what conditions will the MFAM output the library vector closest in Hamming distance to the input? We consider three cases, all of which assume homogeneous nonlinearities [i.e., $Z_n(\alpha) = Z(\alpha)$ for all n] and bipolar inputs and libraries. Similar results can be obtained for the binary input–bipolar library and bipolar input–binary library cases. The remaining binary input–binary filter case will not work on the MFAM without modification of the sgn clip (i.e., $\mathbf{h}$ is always non-negative).

A. Strictly Increasing Nonlinearities

Here we require that $Z(x)$ be a strictly increasing function and that the library vectors be separated sufficiently so that

Table I. Correlation Coefficients for the Four Possible Combinations of Bipolar Binary-Input Library

Input Vector \ Library Element	Bipolar	Binary
Bipolar	$\alpha_n = L - 2h_n$	$\lambda_n = (1/2)(S_g + 1) - h_n$
Binary	$\beta_n = (1/2)(L + S_n) - h_n$	$\gamma_n = (1/2)(L - h_n) + (1/4)(S_g + S_n)$

$$\mathbf{f}_p^T\mathbf{f}_q \leq \alpha; \quad p \neq q, \quad (6)$$

where α represents the magnitude of the largest correlation coefficient among the library elements. Note that, if $\alpha = 0$, the library vectors are orthogonal. Let $\alpha_{\max}$ be the maximum correlation between the input and library vectors:

$$\alpha_{\max} = \max_{1 \leq n \leq N} (\mathbf{g}^T\mathbf{f}_n) = L - 2k, \quad (7)$$

where k is the minimum Hamming distance between $\mathbf{g}$ and the library. We will assume there is a unique library vector $\mathbf{f}_{\max}$ that results in this maximum correlation.

As a consequence of our assumptions, it follows that

$$\alpha_n \leq 2k + \alpha; \quad n \neq \max. \quad (8)$$

The input to the hard limiter is

$$\mathbf{h} = \sum_{n=1}^{N} Z(\alpha_n)\mathbf{f}_n,$$

or, in vector component form,

$$h_l = \sum_{n=1}^{N} Z(\alpha_n) f_{nl}$$
$$= Z(\alpha_{\max}) f_{\max,l} + \sum_{n \neq \max}^{N} Z(\alpha_n) f_{nl}.$$

For $\mathrm{sgn}(h_l) = f_{\max,l}$, we require that

$$h_l \underset{f_{\max,l}=-1}{\overset{f_{\max,l}=1}{\gtrless}} 0,$$

or equivalently $h_l f_{\max,l} > 0$.

Since $a < b$ implies that $Z(a) < Z(b)$ and $f_{nl}f_{\max,l} = \pm 1$, we write

$$h_l f_{\max,l} = Z(\alpha_{\max}) + \sum_{n \neq \max}^{N} Z(\alpha_n) f_{nl} f_{\max,l}$$
$$\geq Z(L - 2k) - \sum_{n \neq \max}^{N} Z(\alpha_n)$$
$$\geq Z(L - 2k) - (N - 1)Z(2k + \alpha),$$

where we have used Eqs. (7) and (8). Thus, if

$$Z(L - 2k) > (N - 1)Z(2k + \alpha), \quad (9)$$

the (noniterative) associative memory in Fig. 1 will give the desired result. For orthogonal library elements, the constraint becomes $Z(L - 2k) > (N - 1)Z(2k)$.

We consider two special cases:

1. Odd Power Nonlinearities

$$Z(x) = x^{2Q+1}; \quad Q = 0,1,2, \ldots.$$

From Eq. (9), for a given L, N, α, and Q, the MFAM will produce the correct output if the input Hamming distance obeys

$$k < \frac{L - \alpha(N-1)^{\frac{1}{2Q+1}}}{2\left[(N-1)^{\frac{1}{2Q+1}} + 1\right]},$$

or, for orthogonal library vectors,

$$k < \frac{L}{2\left[(N-1)^{\frac{1}{2Q+1}} + 1\right]}.$$

For $Q = 0$ we are at the equivalent of Hopfield's associative memory neural net for bipolar inputs and library. Here $k < L/2N$ assures one-step convergence.

We also can compute the minimum Q required for proper MFAM operation by solving Eq. (9) for a given L, k, and α. The result is

$$2Q + 1 > \frac{\ln(N-1)}{\ln\left(\frac{L-2k}{2k+\alpha}\right)}$$

for nonorthogonal library elements and

$$2Q + 1 > \frac{\ln(N-1)}{\ln\left(\frac{L}{2k} - 1\right)}$$

for orthogonal elements.

2. *Exponential Nonlinearity*

$$Z(x) = \exp(ax); \quad a \geq 0. \tag{10}$$

This nonlinearity is motivated from Eq. (2). Using Eq. (9), we can find a proper condition for k which guarantees convergence:

$$k < \frac{1}{4}\left[L - \alpha - \frac{\ln(N-1)}{a}\right].$$

From a design point of view, we would require

$$a > \frac{\ln(N-1)}{L - 4k - \alpha}. \tag{11}$$

B. Exponential Nonlinearities

The exponential nonlinearity in Eq. (10) has an additional convergence property that warrants special attention. If there is only one library vector $\mathbf{f}_{max}$ that produces α_{max}, then

$$h(\mathbf{f}_n,\mathbf{g}) - h(\mathbf{f}_{max},\mathbf{g}) \geq 1,$$

where $h(\,)$ denotes the Hamming distance. Equivalently,

$$\alpha_{max} - \alpha_n \geq 2; \quad n \neq \max.$$

From Eqs. (7) and (8), it follows that $(L - 2k) - (2k + \alpha) \geq 2$. Thus, from Eq. (11), we have the weaker convergence criterion

$$a > \tfrac{1}{2}\ln(N-1). \tag{12}$$

This constraint is not parametrized by α or k. Thus, assuming equality in Eq. (12) is sufficient, the nonlinearity $Z(x) = (N-1)^{x/2}$ will always produce the correct output if there is a unique solution.

C. Even Power Nonlinearities

Physics many times dictates nonlinearities of the form $Z(x) = x^{2Q}$; $Q = 1,2,3, \ldots$ (e.g., square law detectors). Such nonlinearities have the disadvantage of emphasizing negative and positive correlations equally. The analysis of the strictly increasing nonlinearity, however, is applicable here if, in lieu of Eq. (6), we require $|\mathbf{f}_p^T\mathbf{f}_q| \leq \alpha$; $p \neq q$. Large negative correlations are then taken into account. Following the analysis, we find that Eq. (9) is still applicable. It then follows that

$$2Q > \frac{\ln(N-1)}{\ln\left(\frac{L-2k}{2K+\alpha}\right)}.$$

The technique is also directly applicable to other even nonlinearities that are strictly increasing for positive argument such as $Z(x) = \exp(a|x|)$ or polynomials in $|x|$ with positive coefficients.

VI. Extension to Continuous Binary Objects

An object $g(x,y)$ is said to be bipolar if it takes on values of only ± 1. For such objects, the Hamming distance can be defined as the percentage of area over which one object differs from the other. If $\mathcal{A}$ denotes the object's pupil area, Eq. (3) is applicable. Here we let

$$L = \int_{\mathcal{A}} dxdy,$$

$$\alpha_n = \int_{\mathcal{A}} g(x,y)f_n(x,y)dxdy,$$

where g and f_n are, respectively, the bipolar input and nth bipolar library object. There are similar expressions for when the input and/or library is binary. Further application of this generalization to results in this paper is obvious.

VII. Conclusion

We have shown that, for binary (bipolar) objects, the matched filter indicates that library element closest to the input in the Hamming sense. The performance of noniterative matched filter associative memories was also analyzed. Sufficient conditions for desired performance were derived for a number of cases.

This work was supported by the SDIO/IST's program in Ultra High Speed Computing administered through ONR in conjunction with Texas Tech University. Les.E. Atlas was also supported by a NSF PYI award.

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Books *continued from page 2840*

The combination of the background material of the first three chapters with the discussion of particular laser experiments in the later chapters provides the reader with a clear understanding of the physics and chemistry of a variety of photochemical and photothermal surface reactions. At the same time the discussion gives a sense of the potential importance of these laser techniques for possible future technological applications. Only in one case, in the opinion of this reviewer, is the background material a bit too detailed relative to what is needed for understanding the laser experiments that follow. Specifically, the chapter on oxidation of surfaces could well stand alone as a text on the subject, independent of the related laser experiments. Throughout the book, numerous references are cited (of the order of 600 in all) making it possible to investigate areas of specific interest in much greater detail. Tables summarizing the results of related experiments in a particular field are included throughout providing an overview without the need to read all the details of each individual experiment. A few laser-assisted wet processing experiments are discussed, most only in passing. Regrettably, laser-enhanced plating is incorrectly attributed to be a photochemical rather than a photothermal effect. Generally, however, the overall content of the chapters is accurate, informative, and well written.

In summary, this book presents a wealth of useful information on the theory and applications of lasers in relation to microelectronic materials with emphasis on silicon related technologies. The numerous potential uses still require additional detailed investigations before they can be successfully incorporated into manufacturing. In spite of many research efforts, Boyd points out that at the present time relatively only a few laser techniques are just beginning to be accepted to a varying degree in the microelectronics industry. Examples include customization of special circuits, direct writing of waveguide structures, and circuit and mask repair. This gradual acceptance stands in sharp contrast to the extensive incorporation of high-power lasers in many industries for cutting, welding, and surface processing.

In spite of the slowly evolving acceptance and transfer of emerging laser techniques to the semiconducting industry, Boyd gives an optimistic outlook for the future use of lasers in microelectronic materials processing. The field is still relatively young, but there continues to be widespread and increasing interest in its development. This book is a welcome addition to the growing literature on laser processing. It also serves as a worthwhile text for those interested in the more general aspects of the physics and chemistry of laser interactions with insulating and semiconducting materials.

R. J. VON GUTFELD

Photoacoustic and Photothermal Phenomena. Edited by P. HESS and J. PELZL. Springer-Verlag, Berlin, 1988. 573 pp. $65.00.

This book—Vol. 58 of Springer Series in Optical Sciences—is the proceedings of the Fifth International Topical Meeting on Photoacoustic and Photothermal Phenomena, held at the University of Heidelberg, 27–30 July, 1987. This conference is the fifth in the series, held every 2 years, and was truly international with over 230 scientists representing 30 different countries.

The proceedings contain 12 invited talks, 6 progress reports, and 137 selected contributed papers on fundamental investigations of photoacoustic and photothermal phenomena and their applications in physics, chemistry, material science, biology, and medicine. The proceedings are divided into ten parts: spectroscopy; kinetics and relaxation; trace analysis; surfaces and thin films; applications to semiconductors; ultrasonic detection and characterization; mass and heat transfer; thermal wave nondestructive evaluation; experimental techniques; biological and medical applications. The largest section deals with thermal wave nondestructive evaluation, attesting to the power of these techniques for evaluation of subsurface thermal, optical, and magnetic properties.

It is obviously impossible to review each paper here, and, therefore, this reviewer has chosen one paper from each section that seemed particularly novel or timely to him. But before discussing the technical papers, it is appropriate to mention the interesting introductory article entitled Thermal Wave Spectroscopies—Where Do They Win by **R. M. Miller**. He points out that the publication rate of papers in photoacoustic and photothermal spectroscopy is currently about 200–250 papers per year, indicating the usefulness of the techniques. The question raised is: Are the techniques still dominated by developments in the methodology rather than applications by a broad range of users. Miller contends that photoacoustic and photothermal spectroscopies still have not made the transition from a field of experimental study to a routinely available tool. One possible reason for this is that many of the uses to date for the methods have been applications that could be achieved with other more conventional techniques. Miller argues, and this reviewer agrees, that these techniques will flourish when the applications demonstrated are unique rather than being alternatives to conventional spectroscopies.

In the paper by **T. Masujima** on Photoacoustic X-Ray Absorption Spectroscopy, the author used x rays from a synchrotron to excite acoustic waves in materials which were then detected by a micro-

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