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A Class of Continuous Level Neural Nets

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ABSTRACT

A neural net capable of restoring continuous level library vectors from memory is considered. The vectors in the memory library are used to program the neural interconnects. Given a portion of one of the library vectors, the net extrapolates the remainder. Sufficient conditions for unique convergence are stated. An architecture for optical implementation of the network is proposed.

INTRODUCTION

Neural network content addressable memories (CAM's)[1] have stirred great interest in the signal processing community. Such networks have been implemented both optically[1-3] and electronically[4].

The neural net introduced in this paper allows for library vectors with continuous level elements. This is in contrast to previous proposed CAM's where binary library vectors are stored. Initially known neural states are imposed on the net during each iteration. That is, the known states act as the net stimulus and the remaining nodes catalog the response. A human memory analogy is our ability to recall a well known painting by continuously viewing only a portion of it.

PRELIMINARIES

Consider a neural net with L nodes. The transmission from the k^{th} to the i^{th} node is t_{ik}. We will assume a symmetric net ($t_{ik} = t_{ki}$) and will allow for auto-interconnects ($t_{kk} \neq 0$). The state, s_k, of the k^{th} node will be assumed to be a function of the sum of its inputs. For synchronous operation (i.e. all delays between node pairs are identical), we have at time M

$$\vec{i}_{M+1} = \underline{T}\,\vec{s}_M \qquad (1)$$

where $\vec{s}_M$ is a vector of the L neural states at time M, $\vec{i}_M$ is the vector of the L input sums at time M and $\underline{T}$ is the matrix of $t_{i,k}$'s. Let **N** denote the node operator that determines the next set of states from the input sum:

$$\vec{s}_M = \mathbf{N}\,\vec{i}_M \qquad (2)$$

Since the state of the k^{th} node depends only on its input sum, N must be a pointwise vector operator. Substituting (1) into (2) gives the state iteration equation

$$\vec{s}_{M+1} = \mathbf{N}\,\underline{T}\,\vec{i}_M \qquad (3)$$

A MEMORY EXTRAPOLATION NET

Consider a set of N continuous level linearly independent vectors of length $L > N$: $\{\vec{f}_n \mid 0 \le n \le N\}$. We form the library matrix

$$\underline{F} = [\vec{f}_1 \mid \vec{f}_2 \mid \ldots \mid \vec{f}_N]$$

and the interconnect matrix

$$\underline{T} = \underline{F}\,(\underline{F}^T \underline{F})^{-1}\,\underline{F}^T \qquad (4)$$

where the superscript T denotes transposition. We divide the nodes into two sets: one in which the states are known and the remainder, in which the states are unknown. This partition may change from application to application. Without loss of generality, assume that the states of neurons 1 through $P < L$ are known for some vector $\vec{f}$ in the library. Define the node operator by

$$\mathbf{N}\,\vec{i} = \mathbf{N}\,[i_1, i_2, \ldots, i_P \mid i_{P+1}, \ldots, i_L]^T$$
$$= [f_1, f_2, \ldots, f_P \mid i_{P+1}, \ldots, i_L]^T \qquad (5)$$

where f_k is the k^{th} element of $\vec{f}$. That is, for $1 \le k \le P$, the node state is kept at the known value f_k, otherwise, the node state is the input sum. $Q = L - P$ floating neurons will converge to the remainder of f if the first p rows of $\underline{F}$ form a full rank matrix[5].

A TABLE LOOK-UP NET

A **table look-up net** is one in which the same P nodes are always used as the net's stimulus and the remaining Q nodes iteratively converge to the desired response. Note that the iteration in (3) can be partitioned as:

$$\left[\frac{\vec{f}_P}{\vec{s}_{Q,M+1}}\right] = \left[\frac{\underline{T}_P}{\underline{T}_Q}\right]\left[\frac{\vec{f}_P}{\vec{s}_{Q,M}}\right]$$

where $\underline{T}_P$ denotes the first P rows of $\underline{T}$ and $\underline{T}_Q$ is the remaining Q. Since the first P neural states are always clamped to the known values, there is no need to know $\underline{T}_P$. Indeed, an equivalent expression is:

$$\vec{s}_{Q,M+1} = \underline{T}_Q \left[\frac{\vec{f}_P}{\vec{s}_{Q,M}}\right] \qquad (6)$$

A basic methodology for optical implementation of this iteration is illustrated in Fig.1. The known portion of the library vector, $\vec{f}_P$, is input into the processor by an intensity modulated point source array (e.g.

LED's). Multiplication by $\underline{T}_Q$ matrix is performed by a standard vector-matrix multiplication architecture[6]. (The astigmatic optics are not shown). the vector output, $s_{Q,M+1}$, is input into a fiber bundle shown on the right. The bundle is then fed back into the input vector required in (2). We are thus performing the table look-up net at light speed. Feedback could also be provided by mirrors.

The astute reader will have already noted three major problems with this processor:

(1) There is no provision to detect the output.

(2) There is no provision for compensating for absorbtive and other losses in the feedback loop.

(3) The $\underline{T}_Q$ matrix and the input generally contain both positive and negative numbers. Incoherent optics can only add and multiply positive numbers.

Each of these problems has a straightforward solution:

(1) The output can be detected by placing a highly transmitting pellicle in the feedback path and using appropriate focusing optics. This clearly increases absorbtive losses and contributes further to problem two:

(2) If the matrix transmittance can be amplified, then we can compensate for absorbtive loss. One can easily show, for example, if the library vectors are orthogonal, then max (t_{ij}) = N/L. Thus, if L>>N, we can "amplify" the matrix transmittance significantly and still not exceed the maximum passive transmittance value of unity.

(3) The problem of performing bipolar operations with incoherent optics has a number of solutions. One straightforward technique is to rewrite each matrix and vector as the sum of a positive and negative matrix or vector:

$$\vec{f}_p = \vec{f}_p^{\,+} + \vec{f}_p^{\,-}$$

$$\vec{s}_{Q,M} = \vec{s}_{Q,M}^{\,+} + \vec{s}_{Q,M}^{\,-}$$

$$\underline{T}_Q = \underline{T}_Q^{+} + \underline{T}_Q^{-}$$

The matrix $\underline{T}_Q^{+}$, for example, is formed by setting all of the negative elements in $\underline{T}_Q$ to zero. Then (2) can be written as:

$$\vec{s}_{Q,M+1}^{\,+} = \underline{T}_Q^{+}\left[\frac{\vec{f}_p^{\,+}}{\vec{s}_{Q,M}^{\,+}}\right] + \underline{T}_Q^{-}\left[\frac{\vec{f}_p^{\,-}}{\vec{s}_{Q,M}^{\,-}}\right]$$

$$\vec{s}_{Q,M+1}^{\,-} = \underline{T}_Q^{+}\left[\frac{\vec{f}_p^{\,-}}{\vec{s}_{Q,M}^{\,-}}\right] + \underline{T}_Q^{-}\left[\frac{\vec{f}_p^{\,+}}{\vec{s}_{Q,M}^{\,+}}\right]$$

The corresponding optical imple-mentation, although somewhat more involved, requires only positive multiplications and additions and is a straightforward generalization of the architecture in Fig.1. The positive and negative components are added electronically

CONCLUSIONS

Using the **continuous level neural net** (CLNN) algorithm developed in [1], we have proposed a corresponding optical implementation that require no electronics or phase conjugation optics in the feedback path.

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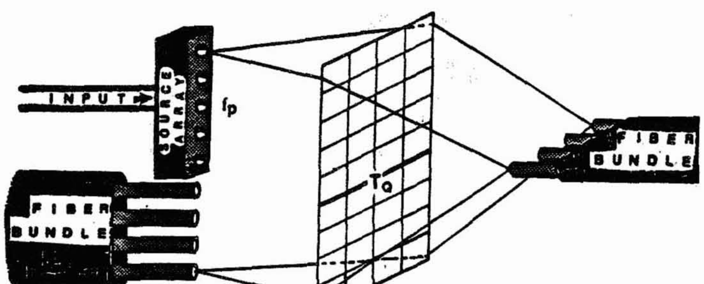

Fig.1