DETECTION OF WEAK SIGNALS USING ADAPTIVE STOCHASTIC RESONANCE

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ABSTRACT

We present a novel nonlinear filtering approach for detecting weak signals in heavy noise from short data records. Such detection problems arise in many applications including communications, radar, sonar, medical imaging, seismology, industrial measurements, etc. The performance of a matched filter detector of a weak signal in heavy noise is directly *proportional to the observation time*. We discuss an alternative detection approach that relies on a nonlinear filtering of the input signal using a bistable system. We show that by adaptively selecting the parameters of the system, it is possible to increase the ratio of the square of the amplitude of a sinusoid to that of the noise intensity around the frequency of the sinusoid (stochastic resonance). The sinusoid can then be reliably detected at the output of the nonlinear system using a suitable matched filter *even when the data record is short.*

1. STOCHASTIC RESONANCE

Stochastic Resonance (SR) is a feature of stochastic relaxation in modulated bistable systems. When the input to such systems is a sinusoidal signal plus an additive observation noise, the noise and signal interact to produce a sharp peak in the power spectrum of the system output at the frequency of the input sinusoid. SR was first introduced by Benzi et al. [1] and has been experimentally observed in various bistable systems of practical importance [2,3,4,5].

The nonlinear system which has been extensively exploited in the study of SR is defined by the nonlinear Langevin equation for one variable[6] as:

$$\dot{x}(t)= ax(t) - bx^3(t) + c\sin(\Omega t) + \xi(t) \qquad (1)$$

where a,b are real parameters, c is the signal amplitude and Ω is the modulation frequency. Here, we assume that the noise $\xi(t)$ is zero mean, Gaussian and white, with an autocorrelation function given by $E[\xi(t)\xi(t+\tau)] = 2D\delta(t-\tau)$.

The system in (1) is the simplest bistable (double-well) system which describes an overdamped Brownian motion in a bistable potential $U(x) = -ax^2/2 + bx^4/4$. The barrier height of the bistable potential in the absence of modulation and noise ($c = 0, D = 0$) is $\Delta U = a^2/4b$ and the potential minima are located at $x_o = \pm\sqrt{a/b}$. With $c > 0$, each potential minimum is alternately raised and lowered relative to the barrier height. Bistability is lost for $c \geq (4a^3/27b)^{1/2}$. Therefore, in the absence of an input ($c = 0, D = 0$) the state of the system is confined to one of the two wells depending on the initial condition. When $c > 0$, the sinusoid induces a periodic variation in the location of the potential minima at its frequency. In this case, the response of the system is of the form, $x_o(t) = \varepsilon\sin(\Omega t + \phi) + x_h(t)$, where $x_h(t)$ represents higher harmonics. The parameters ε and ϕ are normally small compared to a, b and c.

On the other hand, when the noise term $\xi(t)$ is present, noise-driven switching occurs at some rate. The increase in the noise intensity, D, increases the switching rate. The switching rate in the absence of the modulation ($c = 0$) is given by the well known Kramers formula[6]:

$$R = \frac{a}{\pi\sqrt{2}}\exp[-\frac{2\Delta U}{D}] \qquad (2)$$

Transition between wells are therefore more likely when the barrier height is minimum. Since, the sinusoidal signal induces a periodic variation in the location of the potential minima at its frequency, it effectively *clocks* the noise induced transitions. In particular, we will observe a strong sinusoidal component at frequency Ω in $x(t)$ when the noise itself produces on average two transitions per $2\pi/\Omega$ seconds. This provides a simple explanation of SR valid for small driving frequencies Ω.

2. DISCRETE TIME ADAPTIVE STOCHASTIC RESONATORS FOR SIGNAL DETECTION

The performance of a matched filter detector of the signal $A\sin(\Omega t)$ observed in the presence of a Gaussian white noise of intensity σ^2 over an interval of length T seconds is directly proportional to the quantity $A^2T/2\sigma^2$. Therefore, detection performance is poor when T is short and the ratio $(A/\sigma)^2$ is small. To detect weak signals (characterized by a small $(A/\sigma)^2$ ratio) from a short data record, we propose to use an *adaptive stochastic resonator* . Specifically, we pass the noisy observation through the nonlinear bistable filter described by (1). When the parameters of the filter are properly adjusted as we explain below, the output of the filter will have sinusoidal components at frequency Ω and

This work was partially supported by ONR under grant N0014-92-J-1678 and partially by AFOSR under grant AF/F4 9620-93-1-0151 DEF

0-7803-2431-5/95 $4.00 © 1995 IEEE

its odd multiples. In particular, if we denote by A_o the amplitude of the sinusoid at frequency Ω at the output of the filter and by σ^2_{loc} the average intensity of the (1/f) noise in a small band of width proportional to $1/T$ around frequency Ω, we find that the local signal-to-noise-ratio (LSNR)

$(A_o/\sigma_{loc})^2$ is much higher than the input ratio $(A/\sigma)^2$. Therefore, the performance of a detector that uses the output of the bistable filter can be drastically better than that of a detector that uses the noisy observation directly.

2.1. Adaptive selection of the parameters of the stochastic resonator

We begin by explaining how we can select the parameters a and b of (1) for maximum enhancement of the input sinusoid.

Observe that we can normalize (1) by making the change of variables $x(t) \rightarrow x(t)\sqrt{b/a}, t \rightarrow at$ and $c \rightarrow c\sqrt{b/a^3}, D \rightarrow Db/a^2, \Omega \rightarrow \Omega/a$. The normalized eq(1) becomes:

$$\dot{x}(t) = x(t) - x^3(t) + c\sin(\Omega t) + \xi(t). \quad (3)$$

The potential minima in scaled units are now located at $x_o = \pm 1$ and the barrier height $\Delta U = 1/4$. We studied (3) to determine the frequency at which we observe maximum signal enhancement as a function of input noise intensity (SR). We can plot the ratio of the LSNR at the output of the stochastic resonator to that at its input as a function of noise intensity and frequency Ω. These plots show broad maxima for small values of Ω and intermediate value of noise intensities. The curves drop away from their maxima (c.f. [3] and [7] where results corresponding to a particular case of (1) are reported) .

Given this information and a noisy observation of a sinusoid we proceed as follows. We pass an *oversampled* version of the signal through a tree structured filter bank (Fig. 1).

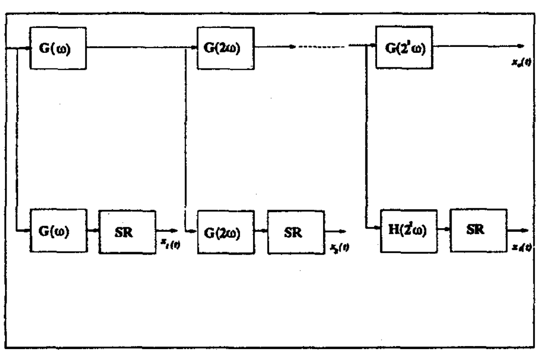

Figure 1: Tree structured stochastic resonator filter bank.

The output of each bank is decimated such that it remains oversampled. In our experiments, we have found that the signal needs to be oversampled at least by a factor of 50 for our technique to work. However, we do not yet have a complete theoretical understanding of the effect of oversampling on our detection scheme.

Next, we compute a crude estimate of the noise intensity by measuring the variance of the filter bank outputs in those bands that do not contain signal energy. Finally, we pass the filter bank output that corresponds to the frequency band that contains the sinusoid through a discrete version of (1) (see below) with parameters a and b adjusted for maximum signal enhancement. The selection of a and b can be done from the coarse knowledge of the noise intensity, the frequency band, plots of output to input SNR corresponding to (3) and the information shown in Figs. 2 and 3. For each input SNR, one must select parameters a and b within particular intervals to achieve maximum enhancement of the LSNR. The vertical bars in the two figures denote these optimal intervals as a function of input SNR. The data shown in these two figures was determined experimentally.

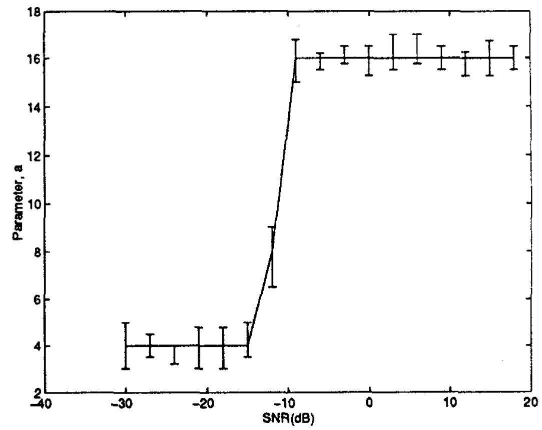

Figure 2: Optimal selection of parameter a in (3) as a function of input SNR.

2.2. Discrete time implementation

Our discrete stochastic resonator corresponds to a fourth order Runge-Kutta discretization of (1). Specifically, if we denote by x_n and u_n the nth samples of $x(t)$ and the input $u(t) = c\sin(\Omega t) + \xi(t)$, our discrete system is described by

$$x_{n+1} = x_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \quad (4)$$

where k_1, k_2, k_3 and k_4 are given by

$$\begin{aligned} k_1 &= h[ax_n - bx_n^3 + u_n] \\ k_2 &= h[a(x_n + \tfrac{k_1}{2}) - b(x_n + \tfrac{k_1}{2})^3 + u_{n+1}] \\ k_3 &= h[a(x_n + \tfrac{k_2}{2}) - b(x_n + \tfrac{k_2}{2})^3 + u_{n+1}] \\ k_4 &= h[a(x_n + k_3) - b(x_n + k_3)^3 + u_{n+2}]. \end{aligned} \quad (5)$$

We set h to be 1/100 of the period of the maximum frequency sinusoid in the band that we consider.

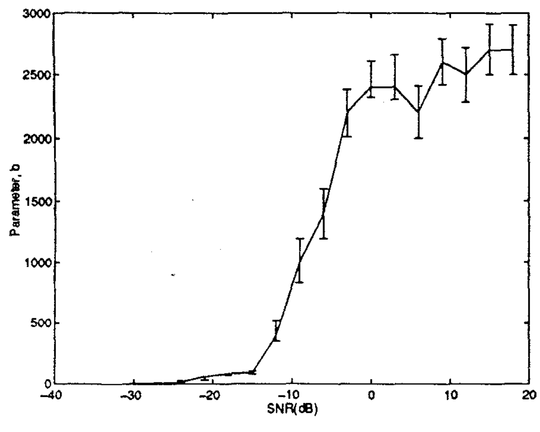

Figure 3: Optimal selection of parameter b in (3) as a function of input SNR.

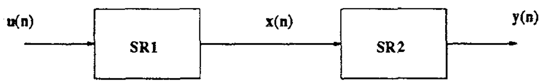

Figure 4: A cascade of two SR devices

3. CASCADED STOCHASTIC RESONATORS

It turns out that it is possible in certain cases to further increase the LSNR by using a cascade of SR devices. In Fig. 4, we consider two such devices connected in series. The two discrete time stochastic resonators SR1 and SR2 in Fig. 4 realize the following system of equations:

$$\begin{aligned} x_n &= a_1 x_n - b_1 x_n^3 + u_n \\ y_n &= a_2 y_n - b_2 y_n^3 + k x_n. \end{aligned} \tag{6}$$

The factor k is varied achieve maximum LSNR at the output of SR2.

We have observed that there is an increase in the LSNR at the output of the second SR device as compared to its input. At the same time we have also observed an increase in the background noise level. We believe therefore that there must be a limit to the amount of LSNR enhancement that one can achieve by cascades of more than two SR devices.

4. EXPERIMENTAL RESULTS

Experimental results of our simulations for the discrete version of SR described in section 2 are presented in Figures 5, 6 and 7. Fig. 8 shows the results corresponding to two cascaded SR devices.

For Fig. 5 and 6 we used $a = 1.0, b = 1.0, c = 0.8$ and $\Omega = 1.0$. The noise levels were $D = 0.245$ and $D = 8.0$ in Fig. 5 and 6 respectively The power spectrum of the output time series was calculated using a FFT. The power spectral density (PSD) was averaged over a very large number of points (2^5 segments of 2^{14} points each). The PSD displayed a series of peaks at odd integer multiples of the modulation frequency. The first peak located at the modulation frequency, however, was the largest. The noise level at the modulation frequency was measured by averaging the values for four points to the left of the signal peak and four points to the right. A strong signal peak appears in the output PSD plots of Figs. 5 and 6 at the modulation frequency. Fig. 6 further shows how the output signal power as well as the LSNR at the modulation frequency are increased with the increase of input noise level. A LSNR increase in the output of a bistable system corresponding to the increase in the input noise level is the prescribing feature of SR.

Fig.7 compares the probability of detection of a sinusoid in white Gaussian noise as a function of input SNR, for the proposed SR approach and classical matched filtering approach that uses the observed signal directly. In the proposed approach, a matched filter detector operates at the output of the SR system. The values of a and b are changed adaptively as a function of the estimated noise intensity using the curves shown in Figs. 2 and 3. The frequency Ω of the input signal was 2 and was assumed to be known. The curves shown are the result of 20 averages at each SNR value.

Finally, Figs. 8.a and 8.b show the output of the first and second SR device respectively for the case where parameters c and Ω were fixed at 0.8 and 1.0, while a_1, b_1 and a_2, b_2 were changed adaptively. It is obvious that the second SR device further improves the LSNR.

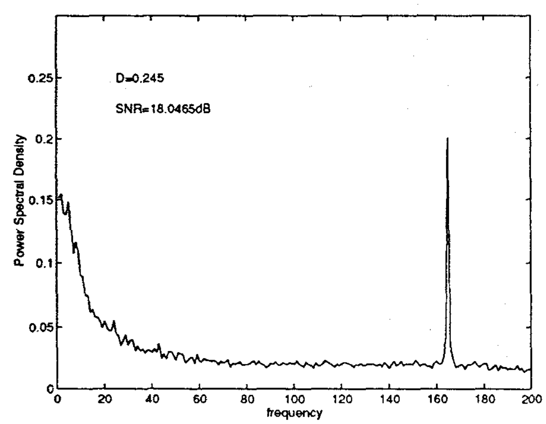

Figure 5: PSD of the output of a SR device with $a = 1.0, b = 1.0, c = 0.8$, $\Omega = 1.0$ and a noise level $D = 0.245$

5. CONCLUSION

In this paper, we studied the problem of detecting a weak signal in the presence of heavy noise from a short data record. We proposed to use an adaptive stochastic resonator to enhance the signal prior to detection. In particular, we showed that the signal-to-noise ratio at the output of a nonlinear bistable filter can be much higher than that at its input when the parameters of the filter are properly

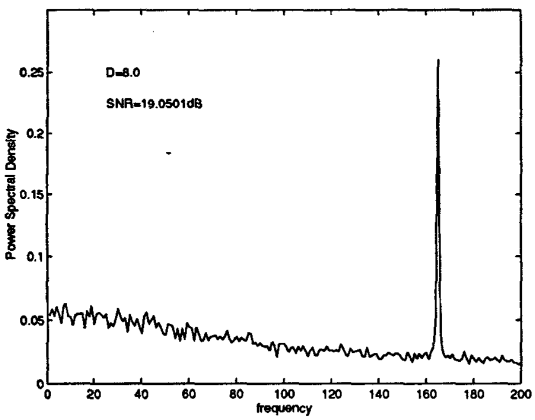

Figure 6: PSD of the output of a SR device with $a = 1.0, b = 1.0, c = 0.8, \Omega = 1.0$ and a noise level $D = 8.0$

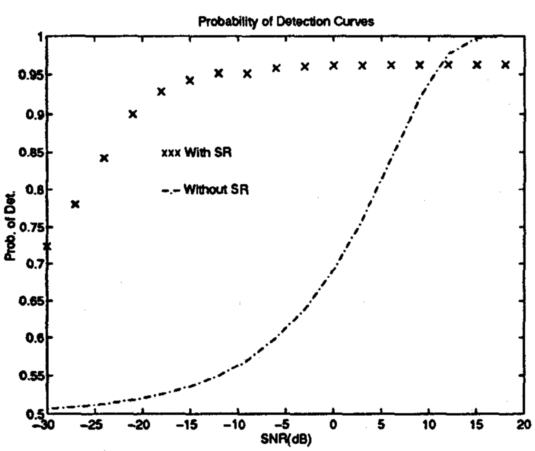

Figure 7: Probability of detection of a sinusoid as a function of input SNT with and without the use of a SR device.

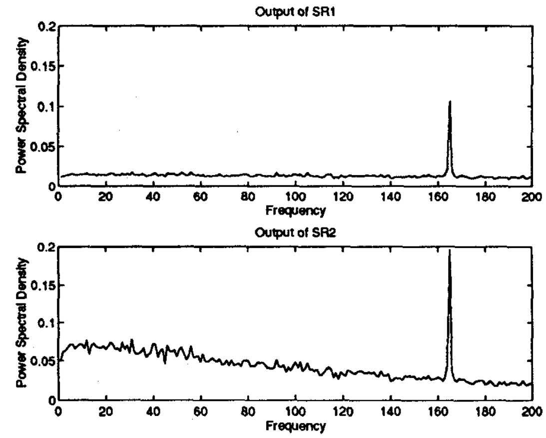

Figure 8: PSDs of the outputs of a cascade of two SR devices.

selected. Although we have concentrated on the detection of sinusoidal signals of known or unknown frequencies in white noise, our approach works equally well with other types of signals, e.g., chirp, PAM, FSK and PWM signals.

6. REFERENCES

[1] R. Benzi, S. Sutera, and A.Vulpiani, *"The mechanism of stochastic resonance,"* J. Phys. A, vol. 14, pp.L453-L457, 1981.

[2] B. McNamara, K. Wiesenfeld, and R. Roy, *"Observation of Stochastic Resonance in a Ring Laser,"* Phys. Rev. Lett., vol. 60, pp.2626-2629, 1988.

[3] G. De-chun, H. Gang, W. Xiao-dong,Y. Chun-yan, Q. Guang-rong, L. Rong, and D. Da-fu, *"Experimental study of signal-to-noise ratio of stochastic resonance systems,"* Phys. Rev. A, vol. 46, pp.3243-3249, 1992.

[4] M. L. Spano, M. Wun-Fogle, and W. L. Ditto, *"Experimental observation of stochastic resonance in a magnetoelastic ribbon,"* Phys. Rev. A, vol. 46, pp.5253-5256, 1992.

[5] V. I. Melnikov, *"Schmitt trigger: A solvable model of stochastic resonance,"* Phys. Rev. E, vol. 48, pp.2481-2489, 1993.

[6] H. Risken, *The Fokker Planck Equation.* Springer-Verlag, Berlin, 1984.

[7] A. S. Asdi, and A. H. Tewfik, *"Detection of weak signals using adaptive stochastic resonance,"* in preparation, to be submitted to *IEEE Trans. on Signal Proc.*, December 1995.