

Mini Quiz
Professor Marks
ELC 5354

March 3, 2015

- Do your work on this page and back.
- Write your name on the back of this page, not the front.
- Show all your work.
- Closed notes and closed books. Two sheets of notes are allowed.

The Erlang random variable with parameters m , λ and α is

$$\Phi_X(u) = \left(\frac{\alpha\lambda}{\lambda + j2\pi u} \right)^m$$

1. What is the value of α ? $\alpha = 1$ so that $\Phi_X(0) = 1$
2. What is the mean of X in terms of m and λ ?
3. What is the variance of X in terms of m and λ ?

$$\Phi_X(u) = m \left(\ln \alpha \lambda - \ln(\lambda + j2\pi u) \right)$$

$$\frac{d\Phi_X(u)}{du} = -m \frac{j2\pi}{\lambda + j2\pi u}$$

$$\frac{d^2\Phi_X(u)}{du^2} = +m \frac{(j2\pi)^2}{(\lambda + j2\pi u)^2}$$

$$\bar{X} = \frac{1}{(-j2\pi)} \frac{-jm2\pi}{\lambda + j2\pi u} \Big|_{u=0} \quad \Phi_X(u) = \int_{-\infty}^{\infty} f_X(x) e^{-j2\pi u x} dx$$

$$= \frac{m}{\lambda} \quad \frac{d\Phi_X}{du} = -j2\pi \bar{X}$$

$$\sigma^2 = \frac{1}{(-j2\pi)^2} m \frac{(j2\pi)^2}{(\lambda + j2\pi u)^2} \Big|_{u=0} = \frac{m}{\lambda^2}$$