Unequal Sameness Theory Addenda

Robert J. Marks II

Here are some additional theorems more firmly establishing USM.

1 The Natural Log of Two is Zero

Theorem: $\ln(2) = 0$

Proof: Consider the series equivalent of ln 2.

$$\begin{aligned}\ln(2) &= 1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \tfrac{1}{5} - \tfrac{1}{6} + \tfrac{1}{7} - \cdots \\ &= \left(1 + \tfrac{1}{3} + \tfrac{1}{5} + \tfrac{1}{7} + \cdots\right) - \left(\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{6} + \tfrac{1}{8} \cdots\right) \\ &= \left(1 + \tfrac{1}{3} + \tfrac{1}{5} + \tfrac{1}{7} + \cdots\right) + \left(\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{6} + \tfrac{1}{8} \cdots\right) - 2\left(\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{6} + \tfrac{1}{8} \cdots\right)\end{aligned} .$$

Adding the first two series gives

$$\begin{aligned}\ln(2) &= \left(1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{6} + \cdots\right) - 2\left(\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{6} + \tfrac{1}{8} \cdots\right) \\ &= \left(1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{6} + \cdots\right) - \left(1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{6} + \cdots\right) . \\ &= 0\end{aligned}$$

and the proof is complete.

2 The logarithm of -1 is zero

Theorem: log(-1) = 0

Proof: Two evaluations of log(-1) follow. The first uses the property that $\log(a^2) = 2\log(a)$.

$$\log\left((-1)^2\right) = 2\log(-1).$$

Since the square of -1 is 1, and the log of one is zero, we can also write

$$\log\left((-1)^2\right) = \log(1) = 0.$$

Equating completes the proof.

3 The Sequential Equality Theorem: All Integers are Equal to Their Next Successive Integer

Theorem:

$$n = n + 1 \tag{1}$$

Proof:

$$(n+1)^2 = n^2 + 2n + 1.$$

Equivalently

$$(n+1)^2 - (2n+1) = n^2.$$

Substract $n(2n+1)$ from both sides and factoring gives

$$(n+1)^2 - (n+1)(2n+1) = n^2 - n(2n+1).$$

To complete the square, add $\frac{1}{4}(2n+1)^2$ to both sides.

$$(n+1)^2 - (n+1)(2n+1) + \frac{1}{4}(2n+1)^2 = n^2 - n(2n+1) + \frac{1}{4}(2n+1)^2.$$

Equivalently

$$\left[(n+1) - \frac{1}{2}(2n+1)\right]^2 = \left[n - \frac{1}{2}(2n+1)\right]^2.$$