Coherent optical extrapolation of 2-D band-limited signals: processor theory

Robert J. Marks II

Given a truncated portion of a band-limited image with known x and y bandwidths, the extrapolation problem is to determine the signal over all x and y. An iterative extrapolation algorithm, recently proposed by Gerchberg, requires only the repeated operations of Fourier transformation and truncation. A coherent optical processor is presented that implements Gerchberg's iterative extrapolation algorithm in two dimensions. Iteration is performed by simple passive feedback.

I. Introduction

The problem of signal extrapolation has long intrigued both mathematicians and engineers. Simply stated, the problem is this: Given a signal in a given signal class over a finite interval, how does one determine the signal over a larger interval? Specific examples of the signal extrapolation problem include prediction, spectrum estimation, and superresolution (resolution beyond the Rayleigh limit). In this paper, a coherent processor is developed that is, in principle, capable of performing extrapolation of 2-D band-limited signals at the speed of light.

The most familiar form of signal extrapolation is the conventional Taylor series. Here, the signal class consists of all functions that are analytic over a given interval. Given such a function over a finite interval, it is possible, in principle, to compute all derivatives of the signal at some interior point. The resulting Taylor series specifies the signal over the entire interval of analyticity. In practice, of course, this method of signal extrapolation is impractical due to the ever-increasing uncertainty in measuring higher-order derivatives.

A second method of extrapolation applicable to the class of band-limited signals has been presented by Slepian and Pollak.[1–2] Here, the known portion of the signal is expanded in an orthonormal series of prolate spheroidal wave functions. The resulting expansion coefficients can then be used in a series expansion for the entire signal. In numerical application, these wave functions are unfortunately most difficult to deal with.[3]

A third, more recent, method of band-limited signal extrapolation has been developed by Gerchberg.[4] Sabri and Steenaart[5] made a digital adaptation of the scheme and obtained some quite remarkable results.

The purpose of this paper is to outline a coherent optical processor implementation of Gerchberg's algorithm for extrapolation of 2-D band-limited signals. Analysis and experimental results will subsequently be reported. In Sec. II, Gerchberg's algorithm is discussed in detail. A coherent processor for performing the algorithm is developed in Sec. III. Section IV contains some concluding remarks.

II. Gerchberg's Algorithm

We here consider the class of low-pass (L_2) band-limited signals. Let $u(x)$ be a signal with Fourier spectrum

$$U(f) = \mathcal{F}[u(x)] = \int_{-\infty}^{\infty} u(x) \exp(-j2\pi f x)dx. \tag{1}$$

Then $u(x)$ is in our signal class if it is square integrable and

$$u(x) = \int_{-W}^{W} U(f) \exp(j2\pi f x)df. \tag{2}$$

The parameter $2W$ is the signal's bandwidth.

Suppose $u(x)$ is known only over a finite interval, say, $-a \le x \le a$. Denote this truncated signal by

$$u_T(x) = u(x)G\left(\frac{x}{2a}\right), \tag{3}$$

where the gate function is defined by

$$G(x) = \begin{cases} 1; & |x| \le 1/2, \\ 0; & |x| > 1/2. \end{cases}$$

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